Distributed Trajectory Estimation with Privacy and Communication Constraints: a Two-Stage Distributed Gauss-Seidel Approach

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Motivation

**Goal:** distributed estimation of trajectories of robots in a team

**Applications:**
- mapping
- exploration
- ...

**why distributed:** avoid exchange of large amount of data
- small flying robots
- underwater vehicles

**Related work:**
- distributed SLAM
  [Dong et al., Paull et al., Bailey et al.]
- multi robot localization
  [Roumeliotis et al., Tron and Vidal]
- distributed optimization
  [Cunningham et al., Nerurkar et al., Franceschelli and Gasparri, Aragues et al.]

**State of the art:** DDF-SAM requires communication cost quadratic in the number of rendezvous.
Cooperative estimation of 3D robot trajectories from relative pose measurements, with the following constraints:

1. Communication only occurs during rendezvous.

2. Data exchange must be minimal (due to limited bandwidth and privacy).

Example application of Privacy Constraint:
Optimization of Multiple trajectories collected through Google Project Tango (courtesy: Simon Lynen)
Contribution

Trajectory estimation as Pose Graph Optimization:

**Related work**: iterative optimization

**Our approach**: 2 stage [Carlone et al. (ICRA 2015)]

- Each phase requires solving a linear system
- We use the Gauss-Seidel algorithm as distributed linear solver
Distributed Gauss-Seidel Approach

\[
\min_{\mathbf{R}_i \in SO(3)} \sum_{(i,j) \in \mathcal{E}} \omega_R^2 \| \mathbf{R}_j - \mathbf{R}_i \bar{\mathbf{R}}_i \|_F^2
\]

\[
\min_{\mathbf{R}_i} \sum_{(i,j) \in \mathcal{E}} \omega_R^2 \| \mathbf{R}_j - \mathbf{R}_i \bar{\mathbf{R}}_i \|_F^2
\]

\[
\min \| A\mathbf{y} - \mathbf{b} \|^2
\]

\[
(A^T A) \mathbf{y} = A^T \mathbf{b}
\]

Hessian (H)

\[
H\mathbf{y} = \mathbf{g}
\]

solve in a distributed manner
Distributed Trajectory Estimation Problem

Hessian Matrix

\[ y^{k+1}_\alpha = H^{-1}_{\alpha\alpha} (H_{\alpha\beta} y^{k}_\beta + g_\alpha) \]

\[ y^{k+1}_\beta = H^{-1}_{\beta\beta} (H_{\beta\alpha} y^{k}_\alpha + g_\beta) \]
Distributed Gauss-Seidel Approach

Trajectory Estimation Problem

$$y_{\alpha_1}$$
$$y_{\alpha_2}$$
$$y_{\alpha_3}$$
$$y_{\alpha_4}$$

robot $\alpha$

$$y_{\beta_1}$$
$$y_{\beta_2}$$
$$y_{\beta_3}$$

robot $\beta$

Hessian Matrix

$$y^{k+1}_\alpha = H^{-1}_{\alpha\alpha} \left( -H_{\alpha\beta} y^k_\beta + g_\alpha \right)$$

$$y^{k+1}_\beta = H^{-1}_{\beta\beta} \left( -H_{\beta\alpha} y^k_\alpha + g_\beta \right)$$

Distributed Gauss-Seidel Approach

Iteration

Error

Centralized
Distributed Gauss-Seidel Approach

Trajectory Estimation Problem

robot $\alpha$

robot $\beta$

\[ y_{\alpha}^{k+1} = H_{\alpha\alpha}^{-1} \left( -H_{\alpha\beta} y_{\beta}^k + g_\alpha \right) \]

\[ y_{\beta}^{k+1} = H_{\beta\beta}^{-1} \left( -H_{\beta\alpha} y_{\alpha}^k + g_\beta \right) \]

Hessian Matrix

distributed Gauss-Seidel

centralized

error

iteration
Distributed Gauss-Seidel Approach

Trajectory Estimation Problem

robot $\alpha$

$y_{\alpha_1}$

$y_{\alpha_2}$

$y_{\alpha_3}$

$y_{\alpha_4}$

robot $\beta$

$y_{\beta_1}$

$y_{\beta_2}$

$y_{\beta_3}$

Hessian Matrix

$H_{\alpha\alpha}$

$H_{\alpha\beta}$

$H_{\beta\alpha}$

$H_{\beta\beta}$

$y^{k+1}_\alpha = H^{-1}_{\alpha\alpha} \left( -H_{\alpha\beta} y^{k}_\beta + g_\alpha \right)$

$y^{k+1}_\beta = H^{-1}_{\beta\beta} \left( -H_{\beta\alpha} y^{k}_\alpha + g_\beta \right)$

Distributed Gauss-Seidel

centralized
Distributed Gauss-Seidel Approach

Trajectory Estimation Problem

\[
y^{k+1}_{\alpha} = H^{-1}_{\alpha\alpha} \left( -H_{\alpha\beta} y^k_{\beta} + g_{\alpha} \right)
\]

\[
y^{k+1}_{\beta} = H^{-1}_{\beta\beta} \left( -H_{\beta\alpha} y^k_{\alpha} + g_{\beta} \right)
\]
Distributed Gauss-Seidel Approach

Trajectory Estimation Problem

Hessian Matrix

\[ y^{k+1}_{\alpha} = H^{-1}_{\alpha\alpha} \left( -H_{\alpha\beta} y^{k}_{\beta} + g_{\alpha} \right) \]

\[ y^{k+1}_{\beta} = H^{-1}_{\beta\beta} \left( -H_{\beta\alpha} y^{k}_{\alpha} + g_{\beta} \right) \]

distributed Gauss-Seidel

centralized
Distributed Gauss-Seidel Approach

Trajectory Estimation Problem

\[ y_{\alpha_{1}} \quad y_{\alpha_{3}} \quad y_{\alpha_{4}} \]

robot \( \alpha \)

\[ y_{\alpha_{2}} \]

robot \( \beta \)

\[ y_{\beta_{1}} \quad y_{\beta_{2}} \quad y_{\beta_{3}} \]

Hessian Matrix

\[ \begin{array}{cccccc}
\alpha_{1} & \alpha_{2} & \alpha_{3} & \alpha_{4} & \beta_{1} & \beta_{2} & \beta_{3} \\
\hline
\alpha_{1} & & & & & & \\
\alpha_{2} & & & & & & \\
\alpha_{3} & & & & & & \\
\alpha_{4} & & & & & & \\
\beta_{1} & & & & & & \\
\beta_{2} & & & & & & \\
\beta_{3} & & & & & & \\
\end{array} \]

\[ H_{\alpha\alpha} \quad H_{\alpha\beta} \]

\[ y_{\alpha_{k+1}} = H_{\alpha\alpha}^{-1} (-H_{\alpha\beta} y_{\beta_{k}} + g_{\alpha}) \]

\[ y_{\beta_{k+1}} = H_{\beta\beta}^{-1} (-H_{\beta\alpha} y_{\alpha_{k}} + g_{\beta}) \]

error

iteration

distributed Gauss-Seidel

centralized
Distributed Gauss-Seidel Approach

Trajectory Estimation Problem

robot $\alpha$

$y_{\alpha_1}$

$y_{\alpha_2}$

$y_{\alpha_3}$

$y_{\alpha_4}$

robot $\beta$

$y_{\beta_1}$

$y_{\beta_2}$

$y_{\beta_3}$

Hessian Matrix

\[
\begin{array}{cccccc}
\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \beta_1 & \beta_2 & \beta_3 \\
\alpha_1 & & & & & & \\
\alpha_2 & & \mathbf{H_{\alpha\alpha}} & & & & \\
\alpha_3 & & & \mathbf{H_{\beta\alpha}} & & & \\
\alpha_4 & & & & \mathbf{H_{\beta\beta}} & & \\
\beta_1 & & & & & \mathbf{H_{\beta\alpha}} & \\
\beta_2 & & & & \mathbf{H_{\beta\beta}} & & \\
\beta_3 & & & & & & \\
\end{array}
\]

\[
y^{k+1}_{\alpha} = H_{\alpha\alpha}^{-1} (-H_{\alpha\beta} y^k_{\beta} + g_{\alpha})
\]

\[
y^{k+1}_{\beta} = H_{\beta\beta}^{-1} (-H_{\beta\alpha} y^k_{\alpha} + g_{\beta})
\]

distributed Gauss-Seidel

centralized

error

iteration
Distributed Gauss-Seidel Approach

Trajectory Estimation Problem

robot $\alpha$

$y_{\alpha_1}$

$y_{\alpha_3}$

$y_{\alpha_4}$

robot $\beta$

$y_{\beta_1}$

$y_{\beta_2}$

$y_{\beta_3}$

$\begin{align*}
y_{\alpha}^{k+1} &= H^{-1}_{\alpha \alpha} \left( -H_{\alpha \beta} y_{\beta}^{k} + g_{\alpha} \right) \\
y_{\beta}^{k+1} &= H^{-1}_{\beta \beta} \left( -H_{\beta \alpha} y_{\alpha}^{k} + g_{\beta} \right)
\end{align*}$

Hessian Matrix

$\begin{array}{cccc}
\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\
\alpha_1 & H_{\alpha \alpha} & & \\
\alpha_2 & & H_{\beta \alpha} & \\
\alpha_3 & & & H_{\beta \beta} \\
\alpha_4 & & & \\
\beta_1 & & & \\
\beta_2 & & & \\
\beta_3 & & & \\
\end{array}$

distributed Gauss-Seidel

centralized

error

iteration
Distributed Gauss-Seidel Approach

Trajectory Estimation Problem

robot $\alpha$

$y_{\alpha_1}$

$y_{\alpha_2}$

$y_{\alpha_3}$

$y_{\alpha_4}$

robot $\beta$

$y_{\beta_1}$

$y_{\beta_2}$

$y_{\beta_3}$

$y_{k+1}^{\alpha} = H^{-1}_{\alpha\alpha} (-H_{\alpha\beta} y_{k}^{\beta} + g_{\alpha})$

$y_{k+1}^{\beta} = H^{-1}_{\beta\beta} (-H_{\beta\alpha} y_{k}^{\alpha} + g_{\beta})$

Hessian Matrix

$H_{\alpha\alpha}$

$H_{\alpha\beta}$

$H_{\beta\alpha}$

$H_{\beta\beta}$

Distributed Gauss-Seidel Approach
Simulation Results

The approach has the following merits:

1. **Proven convergence to centralized. Fast convergence with smart initialization**

2. Communication is linear in number of rendezvous

3. Scalability in the number of robots

4. Resilience to noise
Simulation Results

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Simulation Results

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4. Resilience to noise
Field Experiments

We tested the proposed approach on field data collected by two to four Jackal robots, moving in a military test facility. We use the estimated trajectories to reconstruct a 3D map of the facility.
Field Experiments (4 Robots)
Thank you!

For further information, please come to the interactive session: 1.4 (Balcony)