Distributed Trajectory Estimation with Privacy and Communication Constraints: a Two-Stage Distributed Gauss-Seidel Approach

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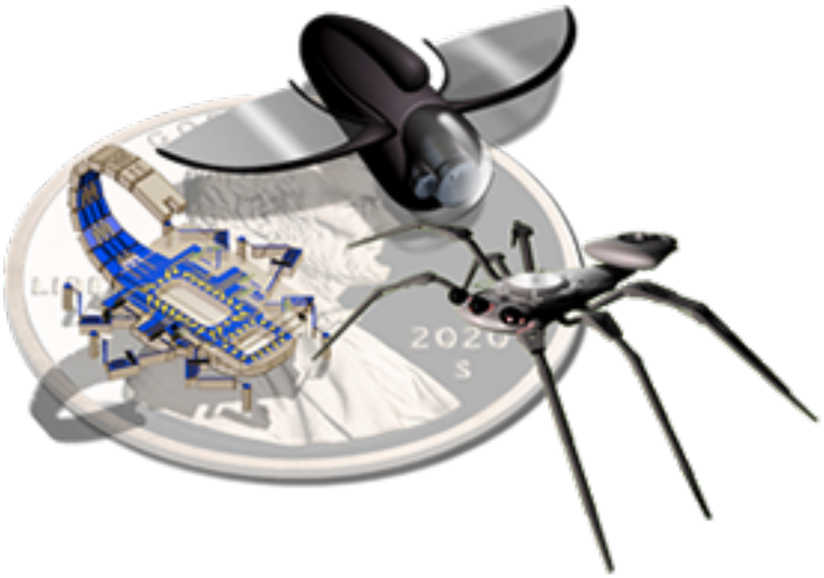
Motivation

- **Goal**: distributed estimation of trajectories of robots in a team
- **Applications**:
  - mapping
  - exploration
  - …
- **why distributed**: avoid exchange of large amount of data
  - small flying robots
  - underwater vehicles



Related work:

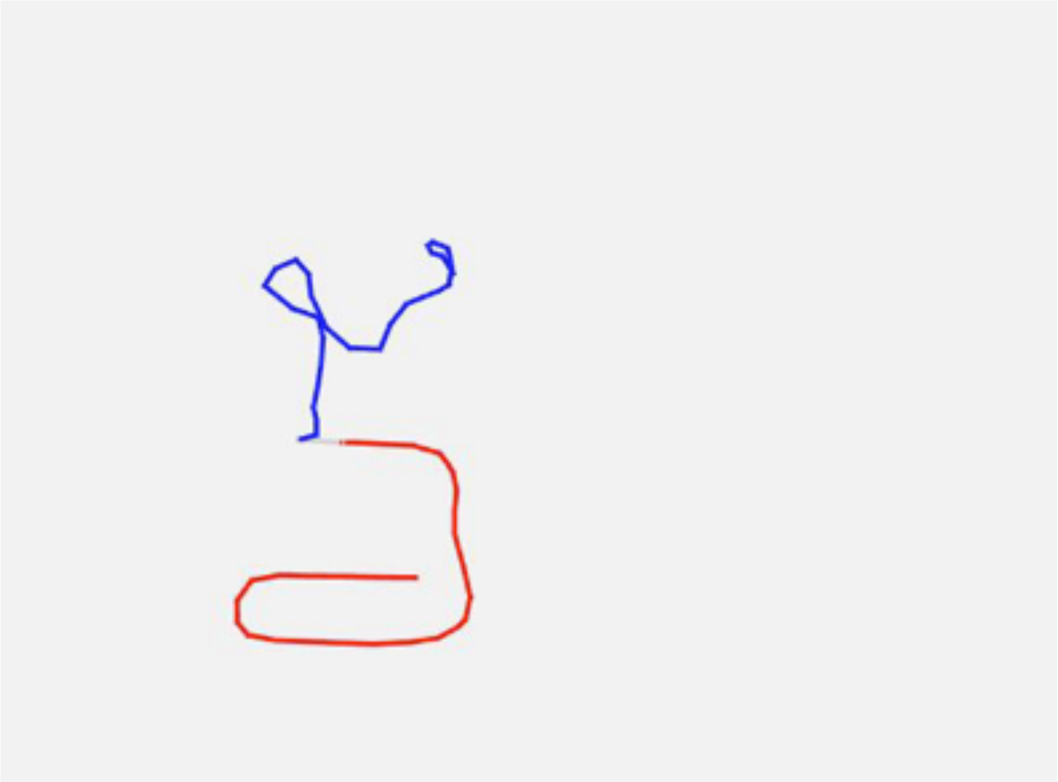
- distributed SLAM
  [Dong et al., Paull et al., Bailey et al.]
- multi robot localization
  [Roumeliotis et al., Tron and Vidal]
- distributed optimization
  [Cunningham et al., Nerurkar et al., Franceschelli and Gasparri, Aragues etl al.]
- ***State of the art***: DDF-SAM requires communication cost quadratic in the number of rendezvous.

Problem Statement

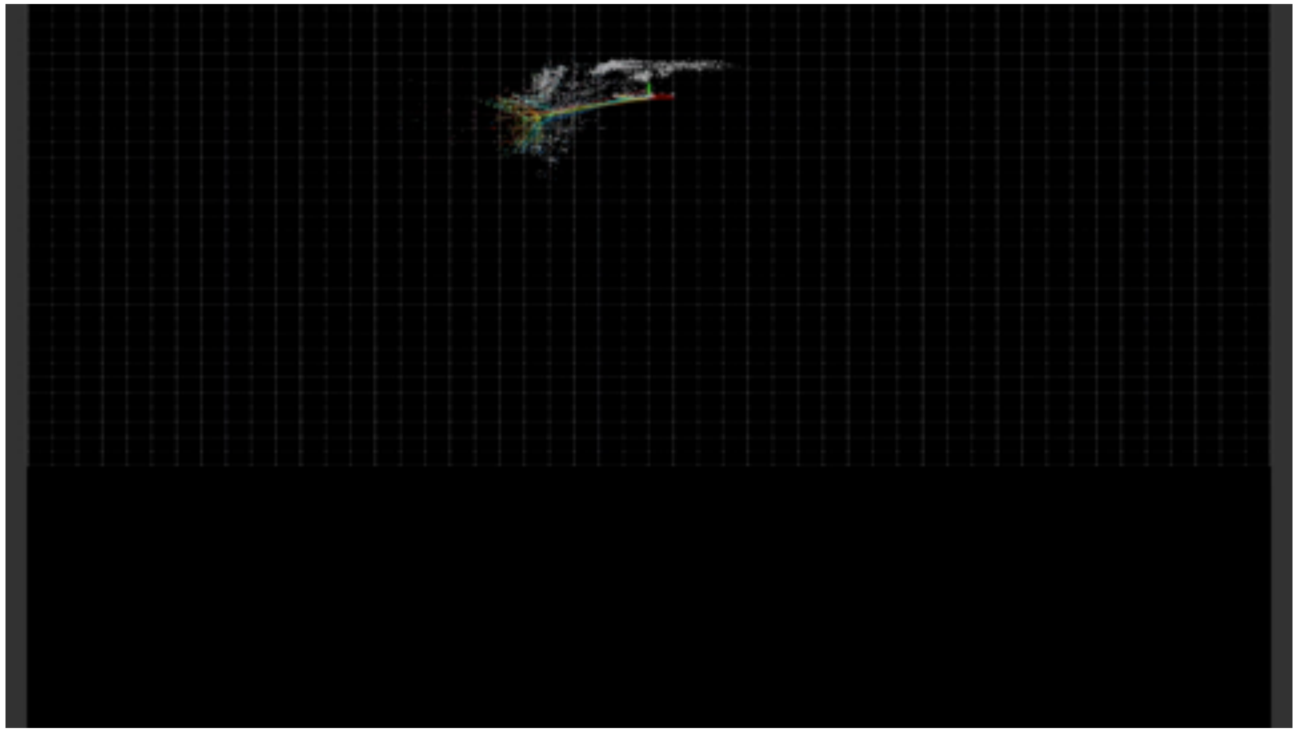
Cooperative estimation of 3D robot trajectories from **relative pose measurements**, with the following constraints:

1. Communication only occurs during **rendezvous**.

2. Data exchange must be minimal (due to **limited bandwidth** and **privacy**).



Communication only occurs when two robots are close enough.

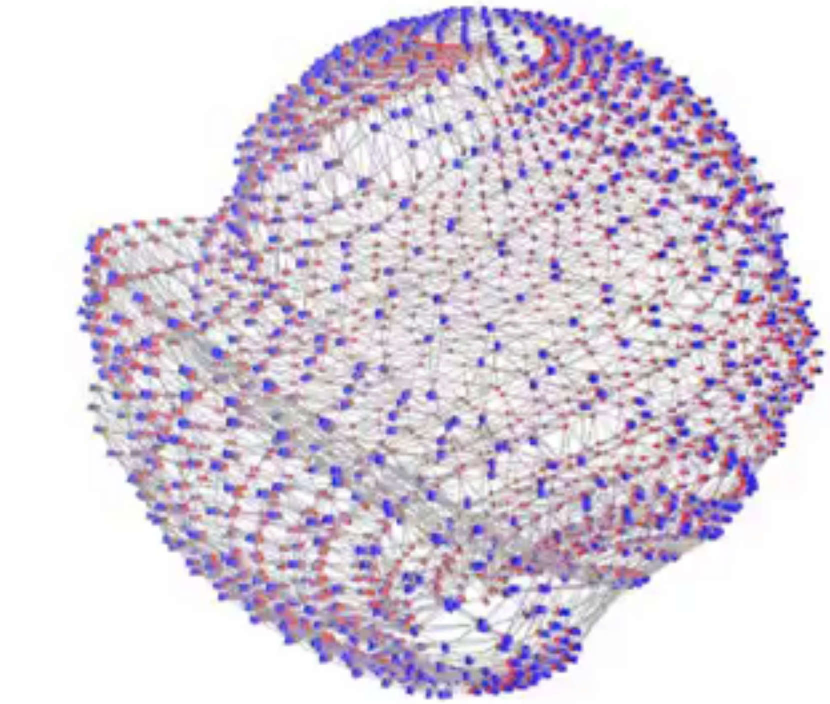


Example application of Privacy Constraint: Optimization of Multiple trajectories collected through Google Project Tango
(courtesy: Simon Lynen)

Contribution

Trajectory estimation as Pose Graph Optimization:

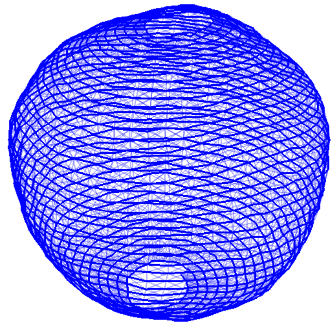
Related work: iterative optimization



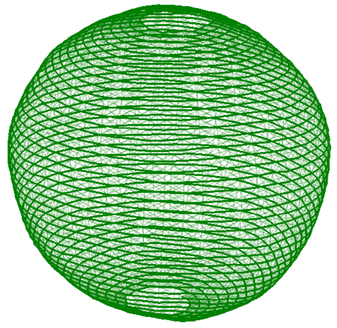
SLAM - TORO - Sphere Optimization
courtesy: Cyril Stachniss

Our approach: 2 stage [Carlone et al. (ICRA 2015)]

- Each phase requires solving a linear system
- We use the Gauss-Seidel algorithm as distributed linear solver



Estimate



Optimum

Distributed Gauss-Seidel Approach

$$\min_{\boldsymbol{R}_i \in \mathrm{SO}(3)} \sum_{(i,j)\in\mathcal{E}} \omega_R^2 \|R_j - R_i \bar{R}_i^j\|_F^2$$

↓ quadratic relaxation

$$\min_{\boldsymbol{R}_i} \sum_{(i,j)\in\mathcal{E}} \omega_R^2 \|R_j - R_i \bar{R}_i^j\|_F^2$$

↓ rewrite

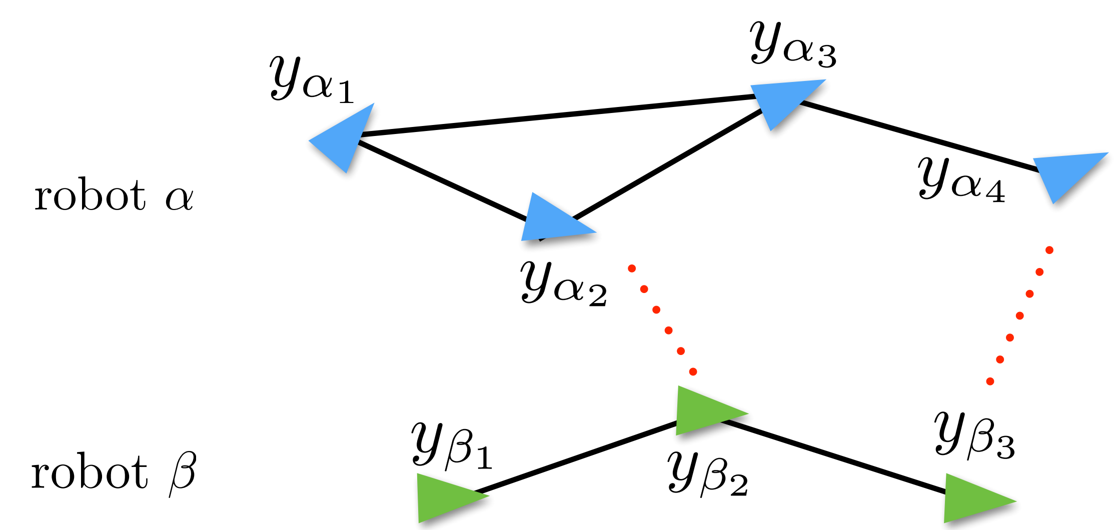
$$\min_y \|Ay - b\|^2$$

↓ normal equation

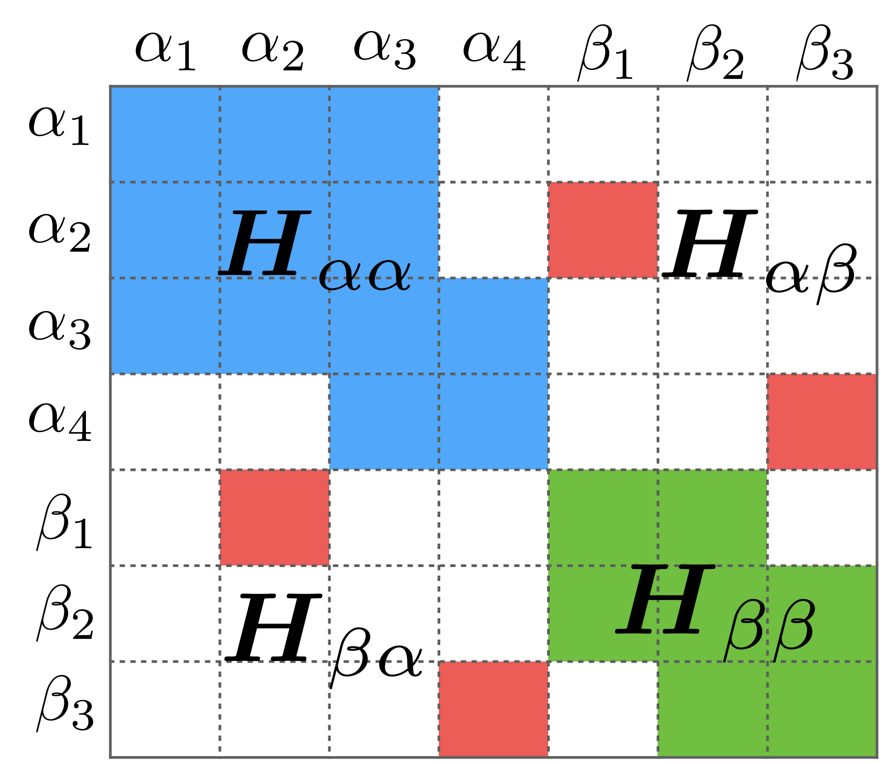
$$\underbrace{\left(A^T A\right)}_{\text{Hessian (H)}} y = \underbrace{A^T b}_{\text{g}}$$

↓

$$Hy = g$$ solve in a distributed manner



Hessian Matrix



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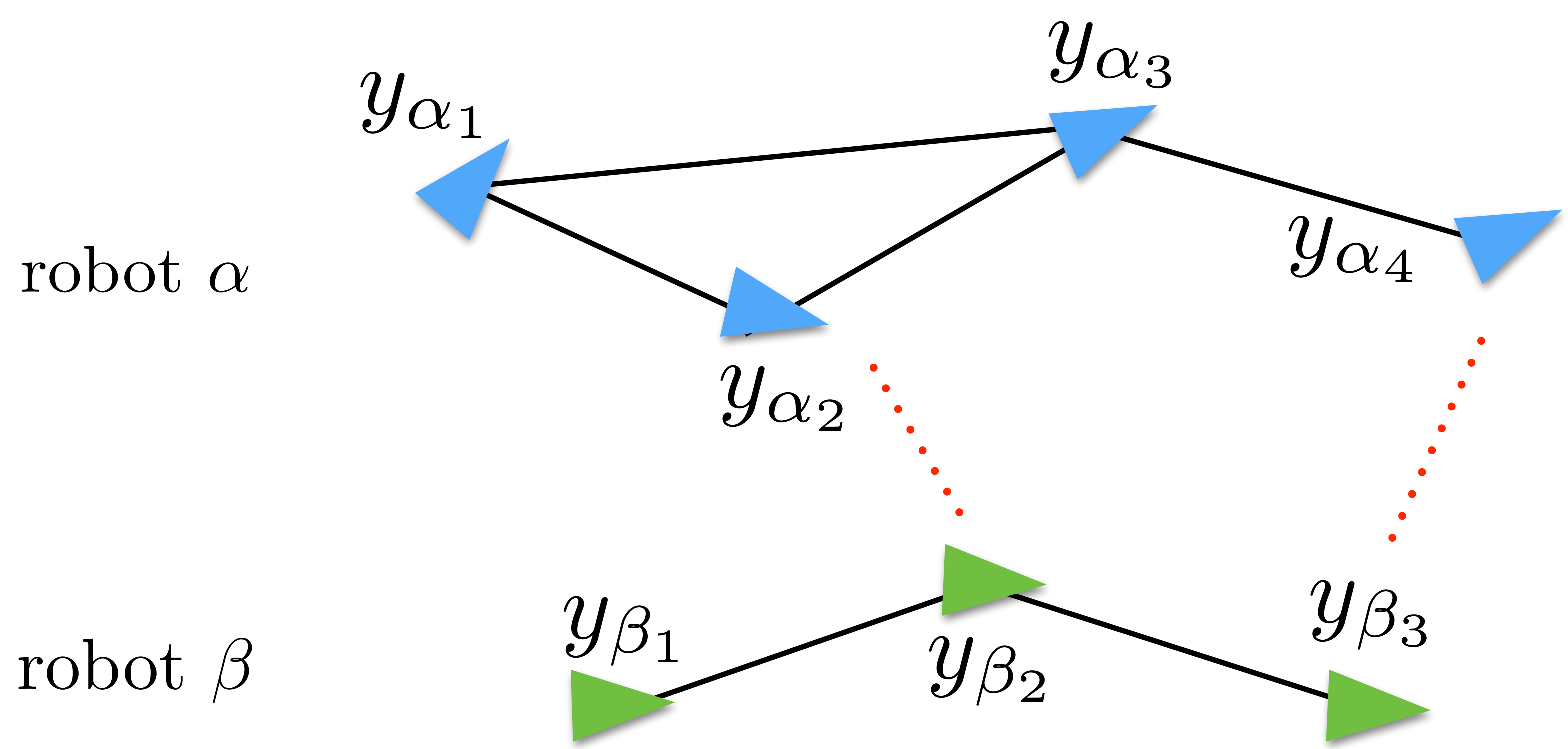
Distributed Gauss-Seidel Approach

Trajectory Estimation Problem



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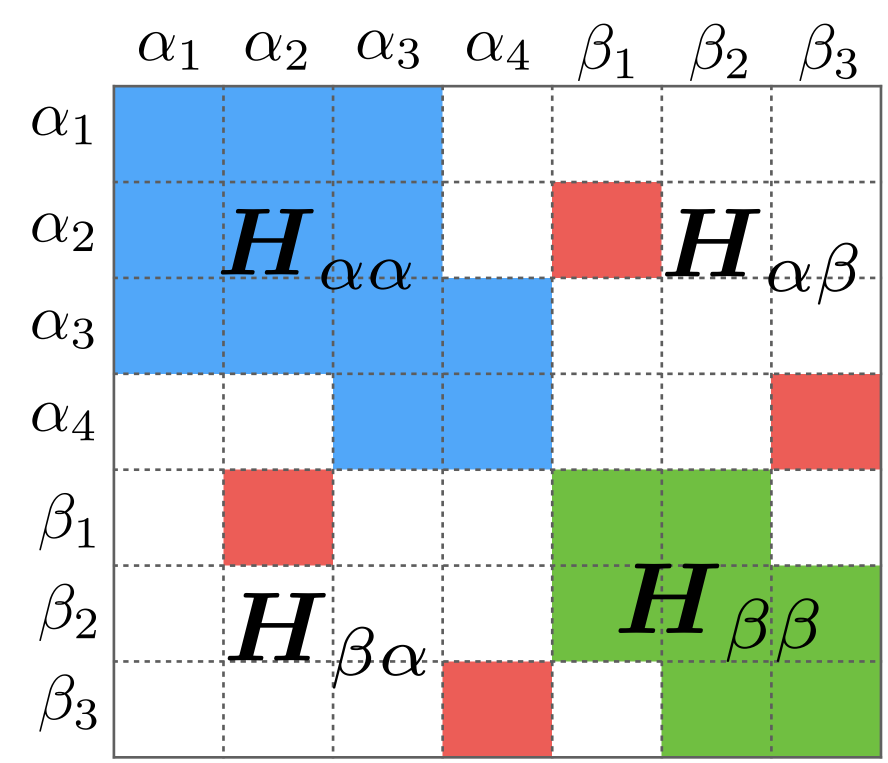

Iterate

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Distributed Gauss-Seidel Approach

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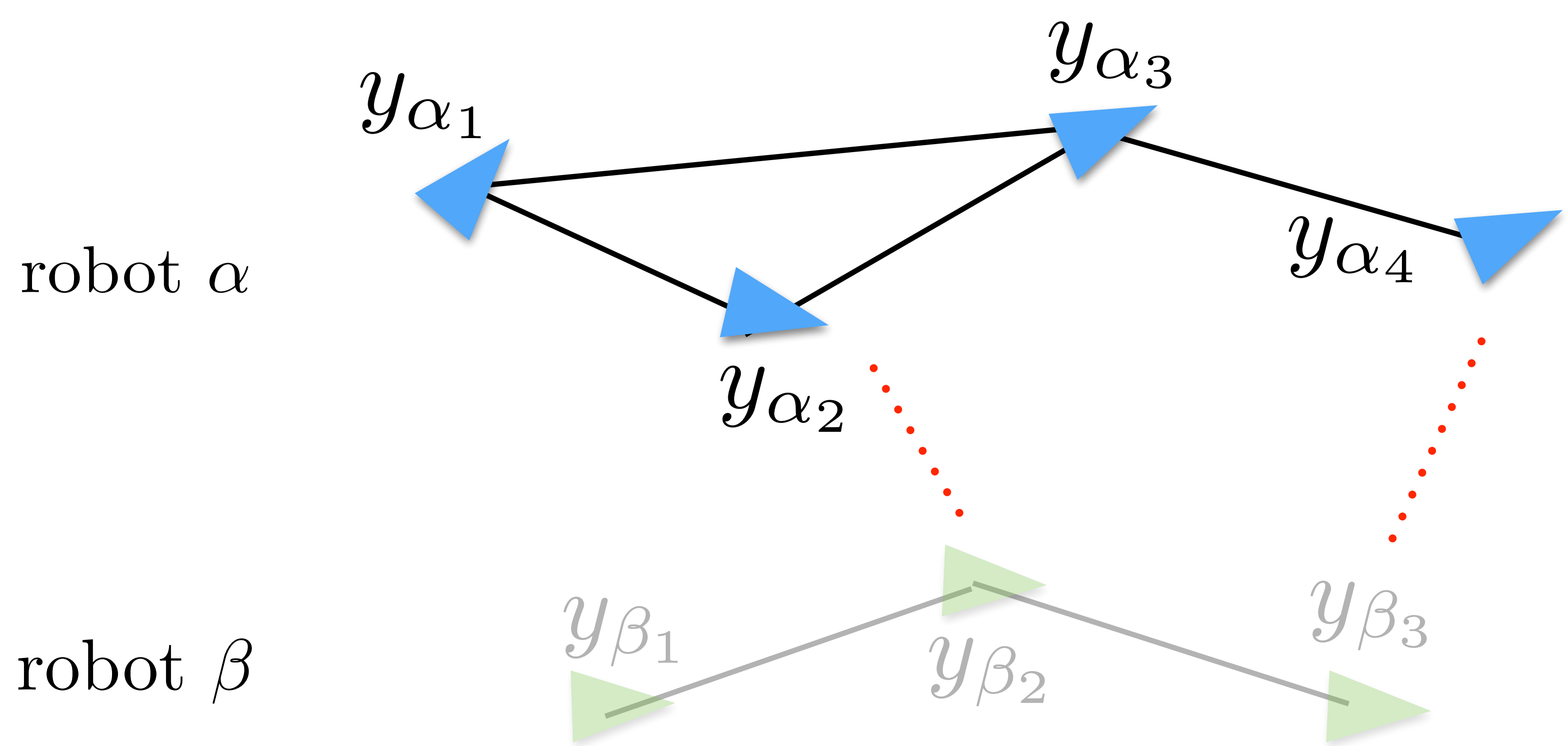

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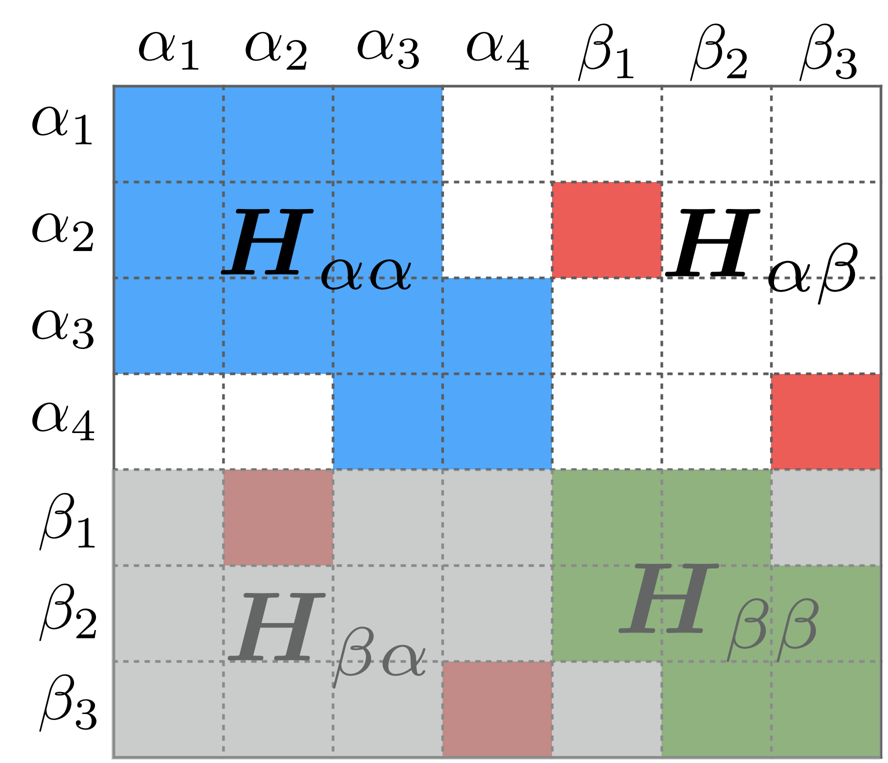

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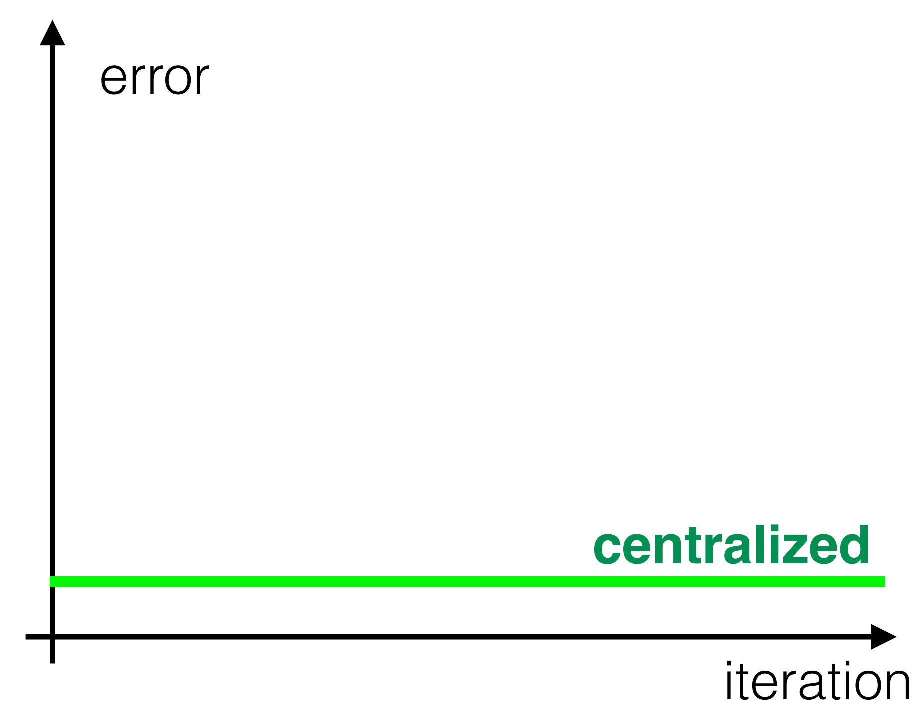

Distributed Gauss-Seidel Approach

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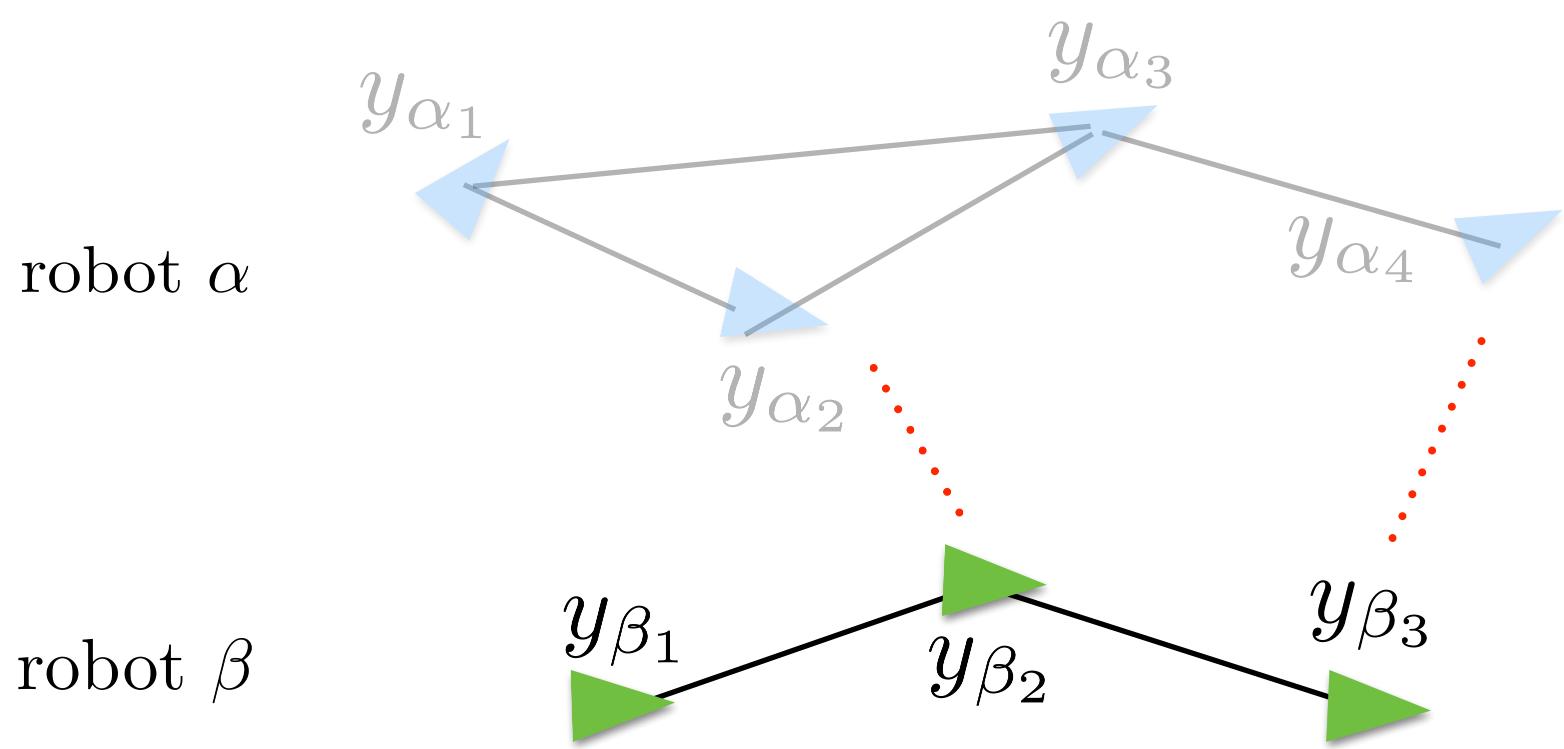

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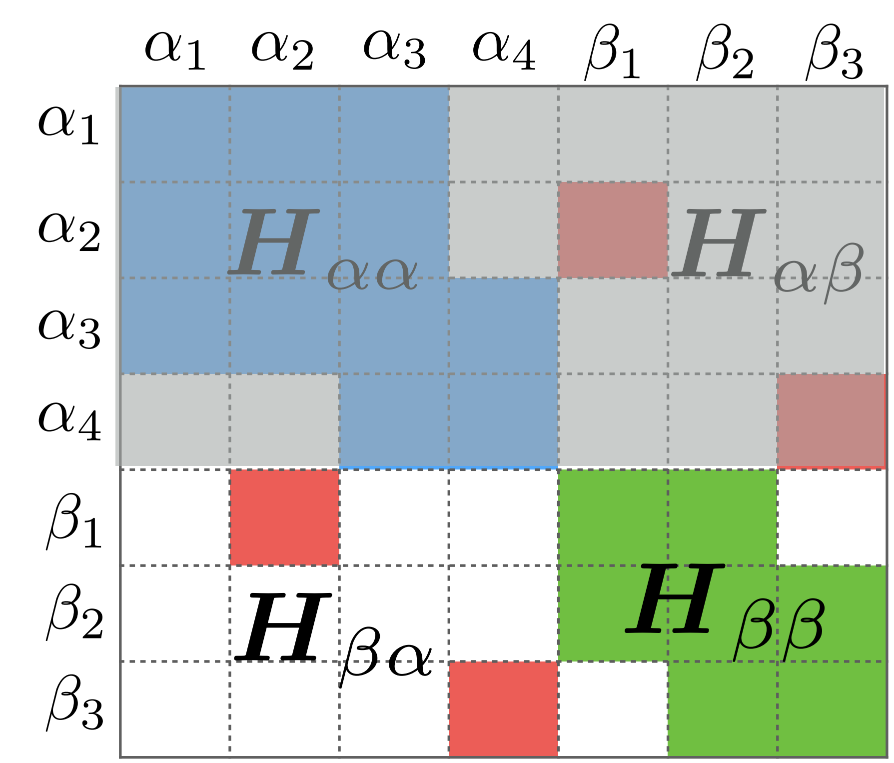

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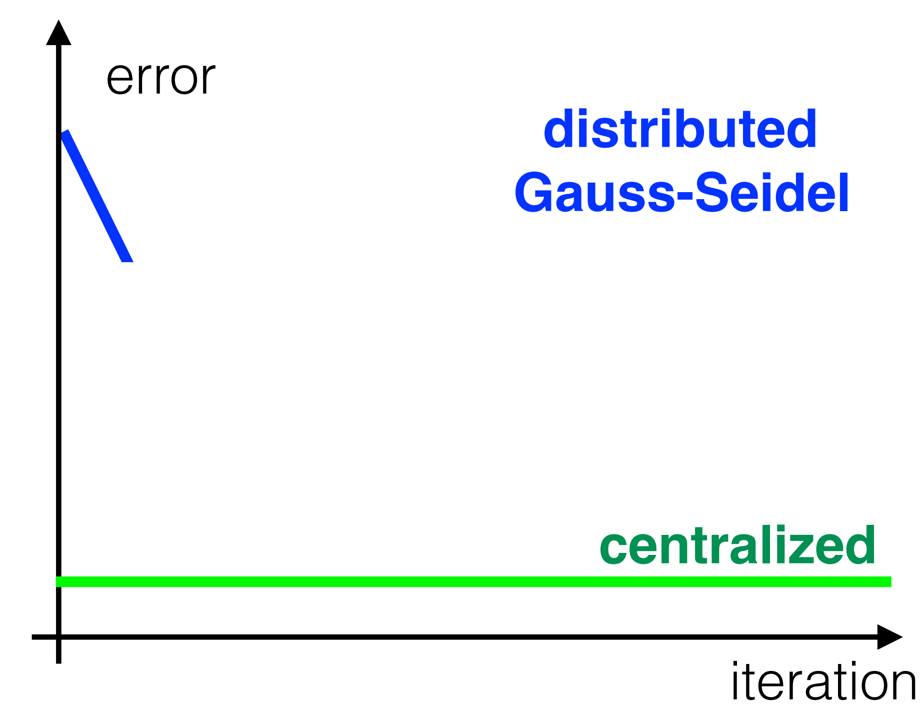

Distributed Gauss-Seidel Approach

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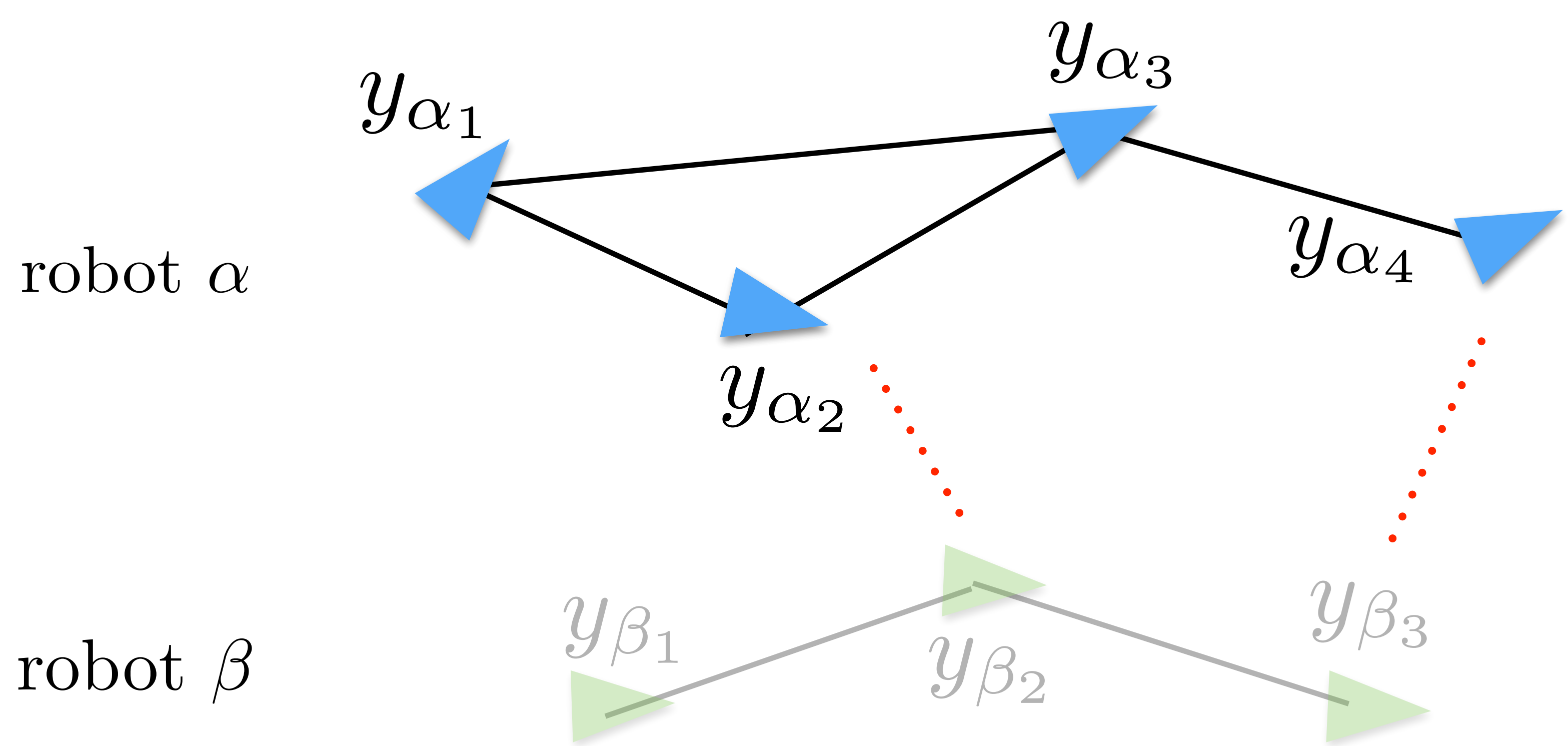

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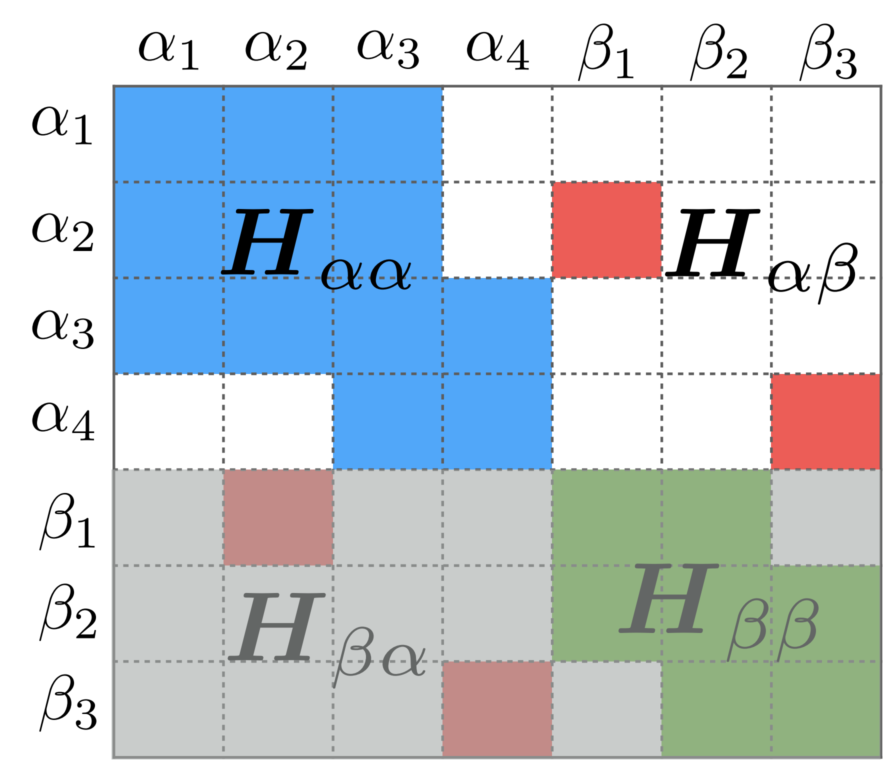

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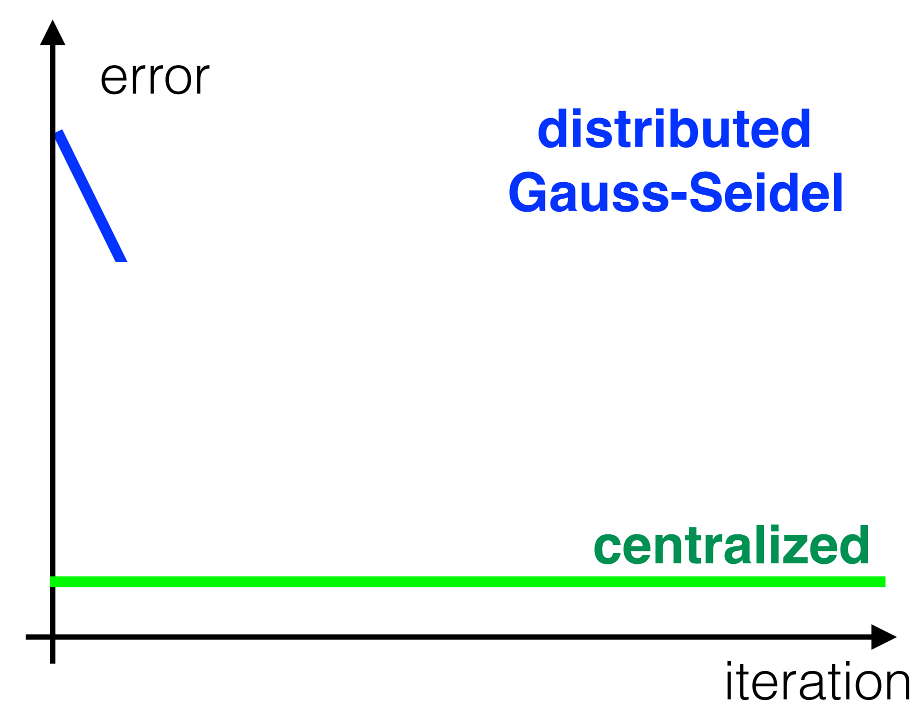

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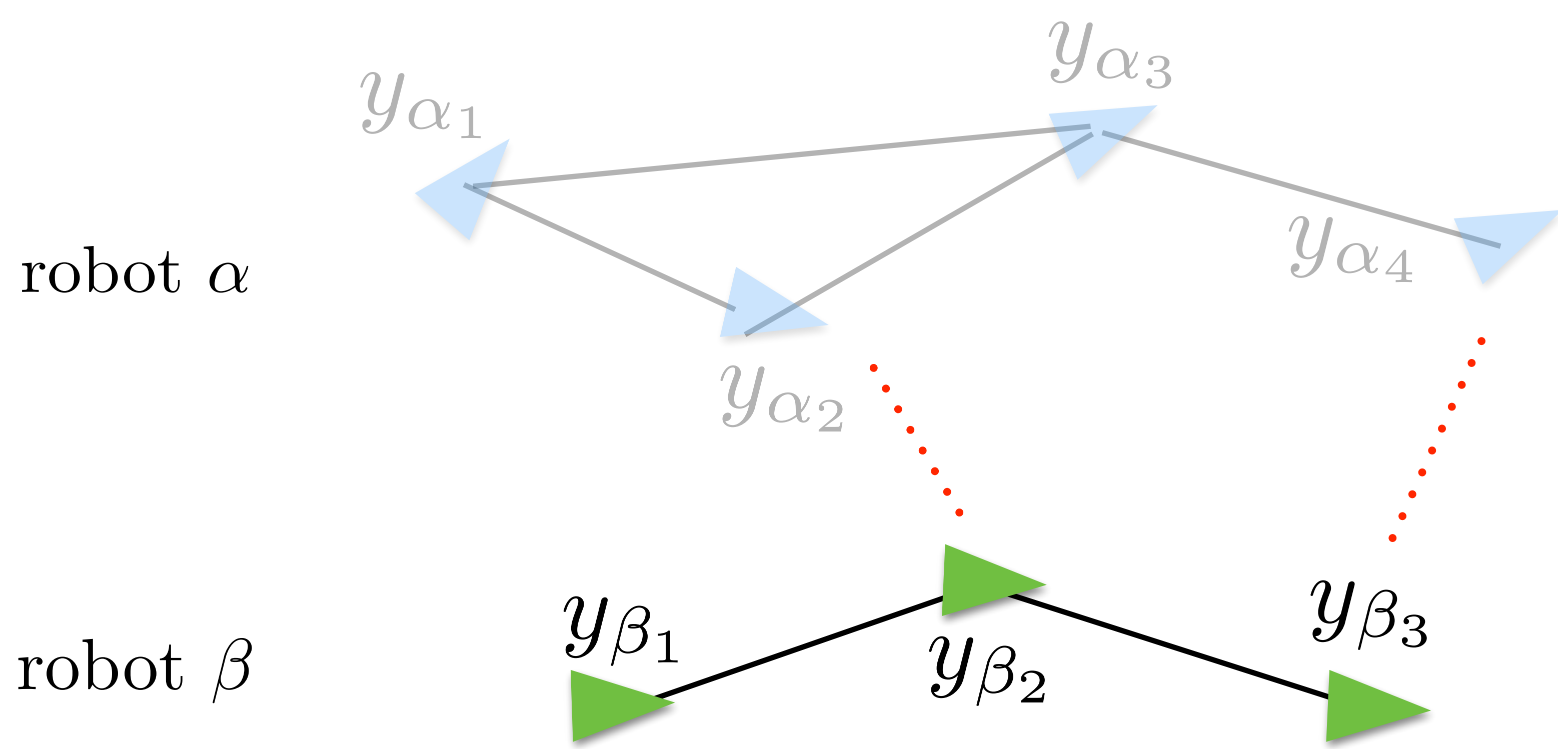

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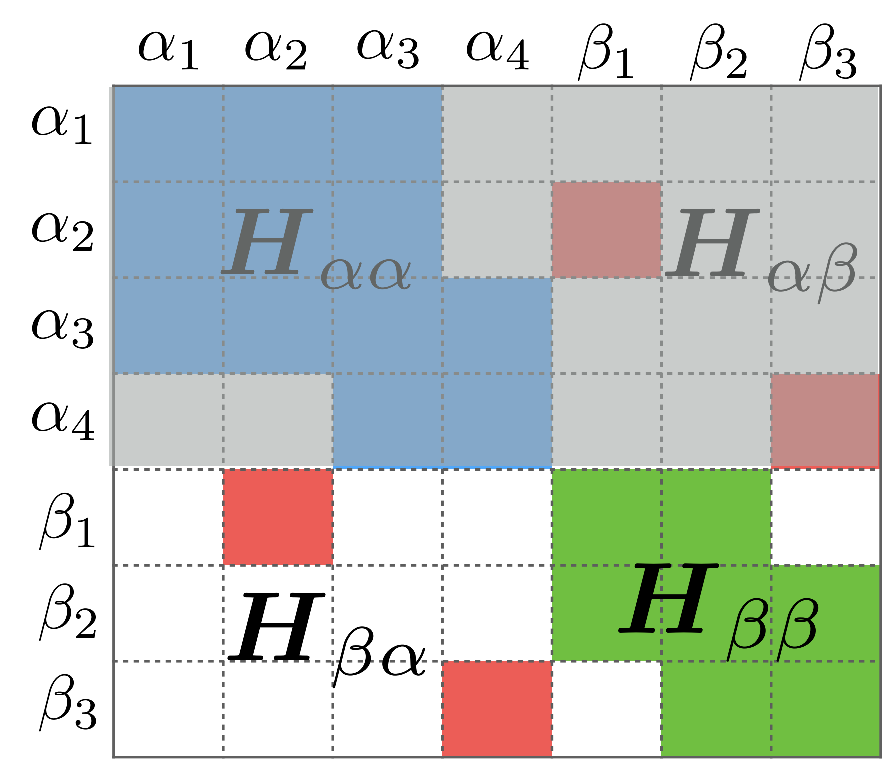

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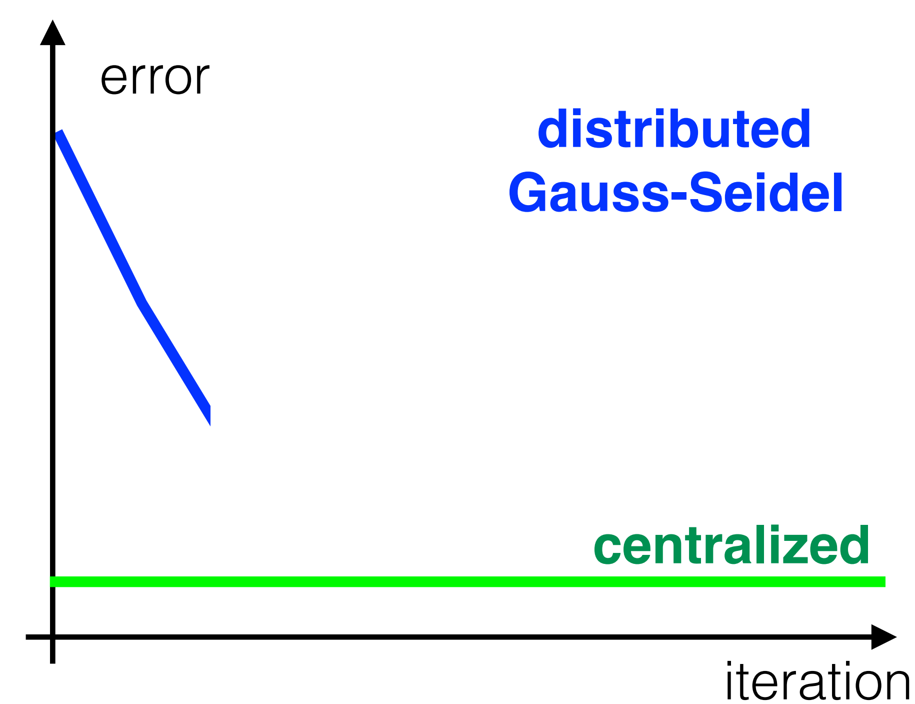

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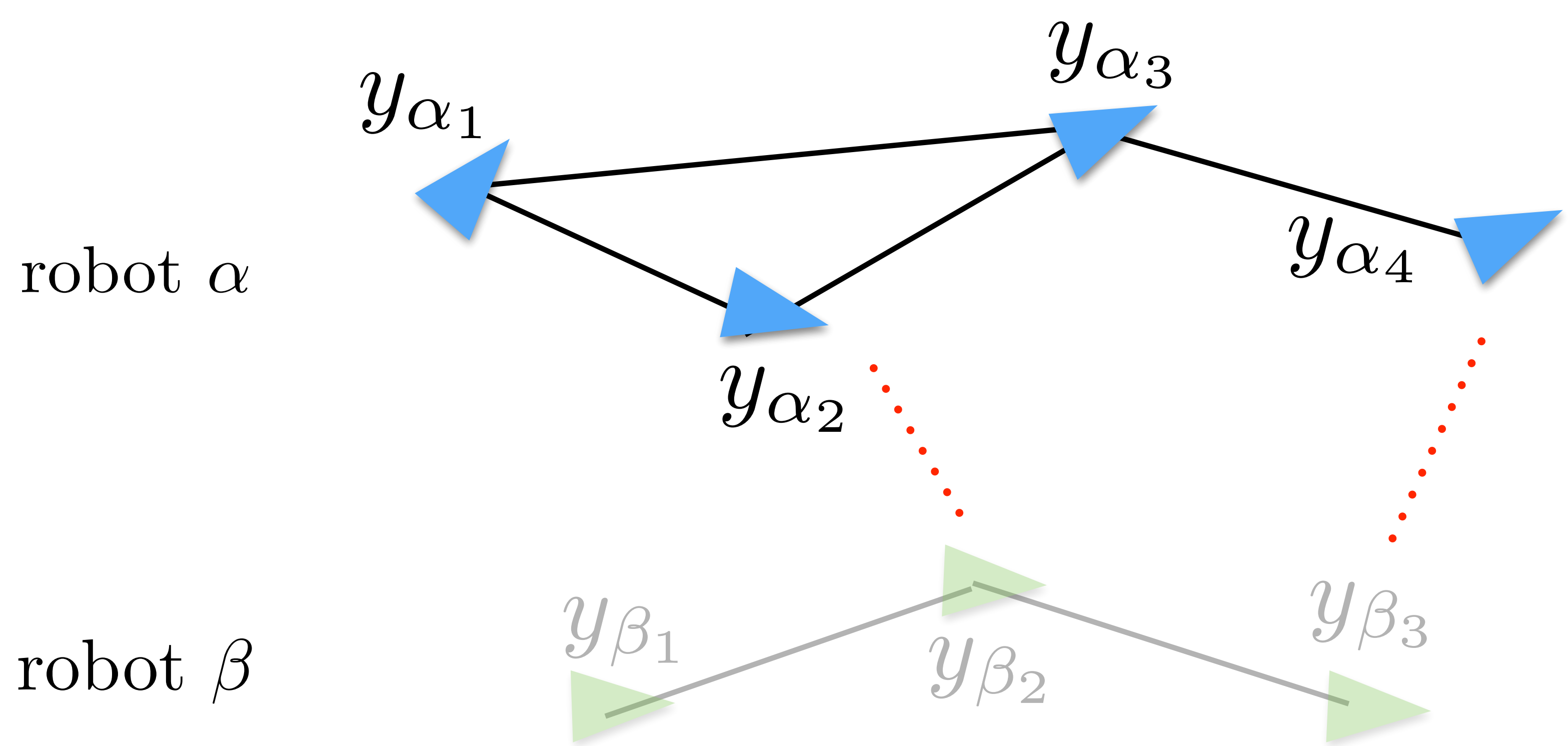

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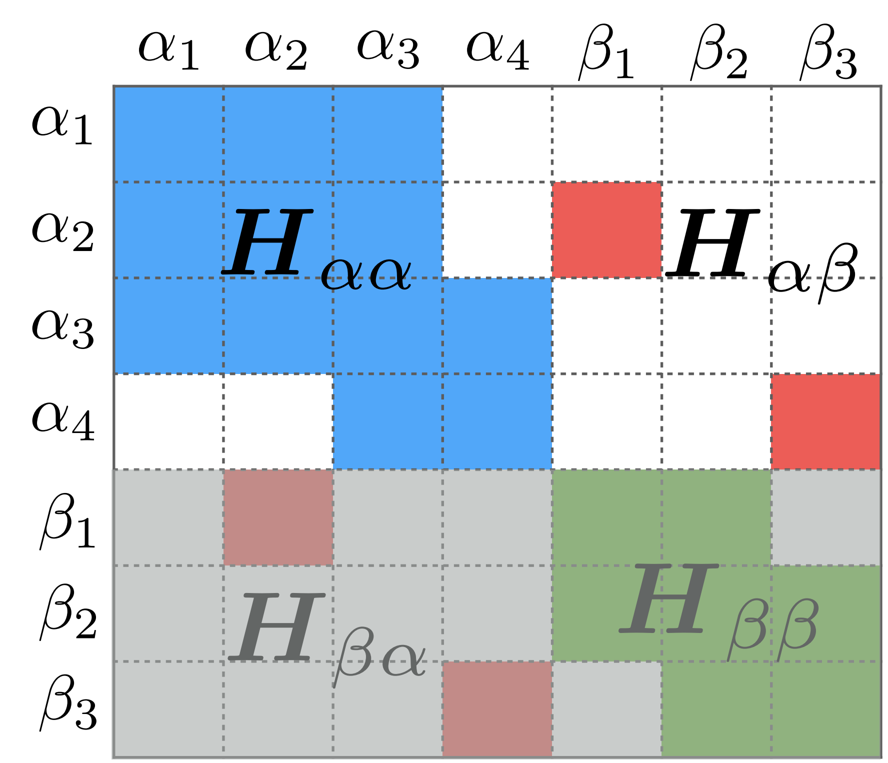

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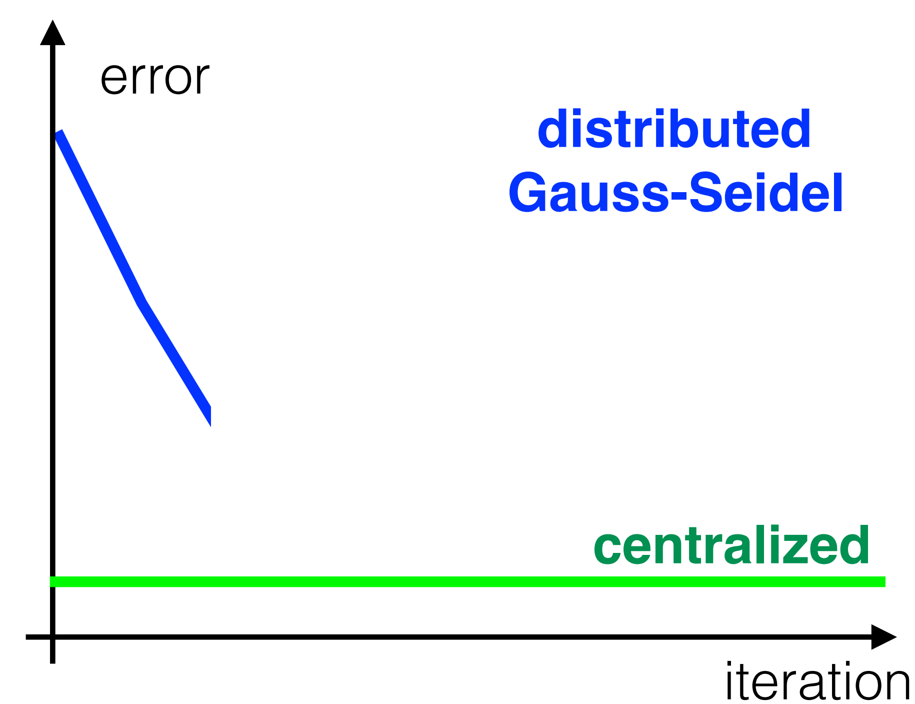

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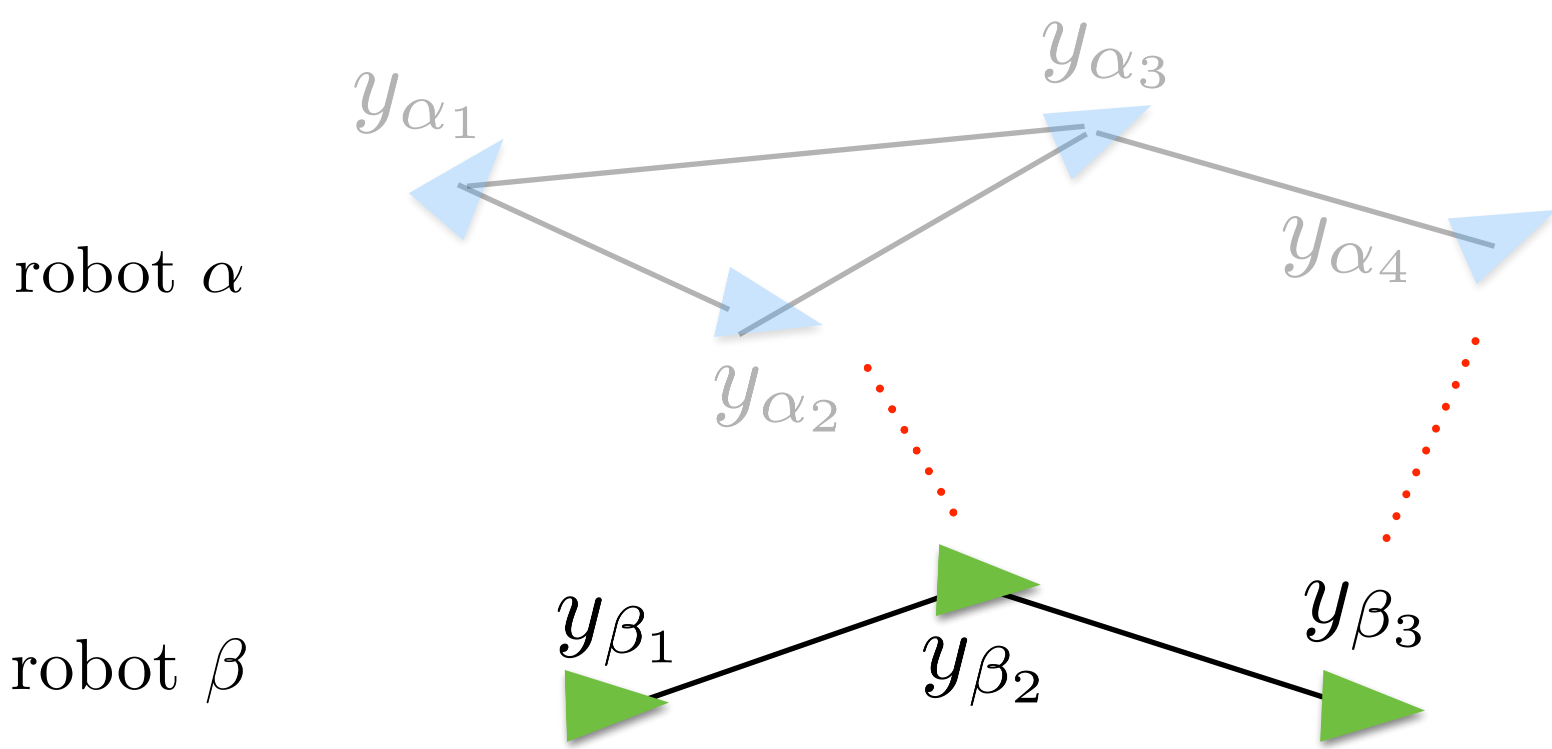

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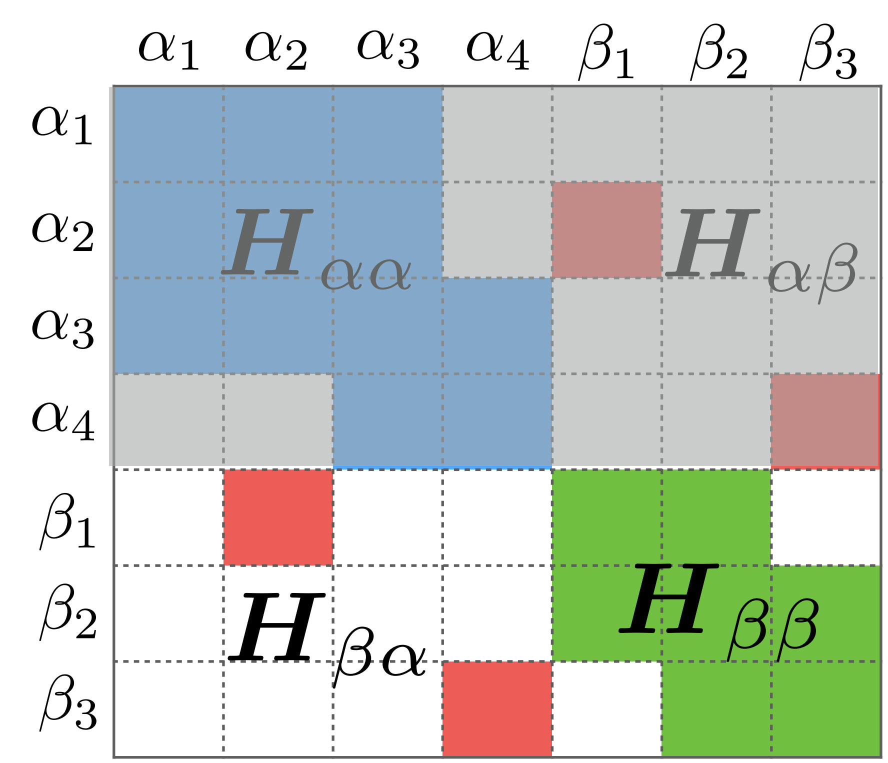

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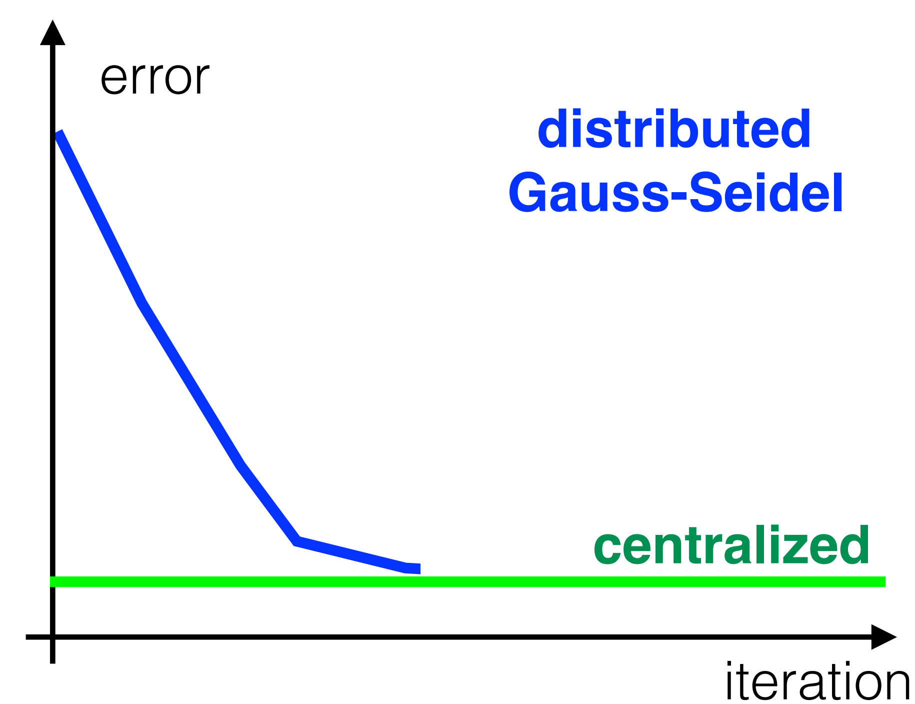

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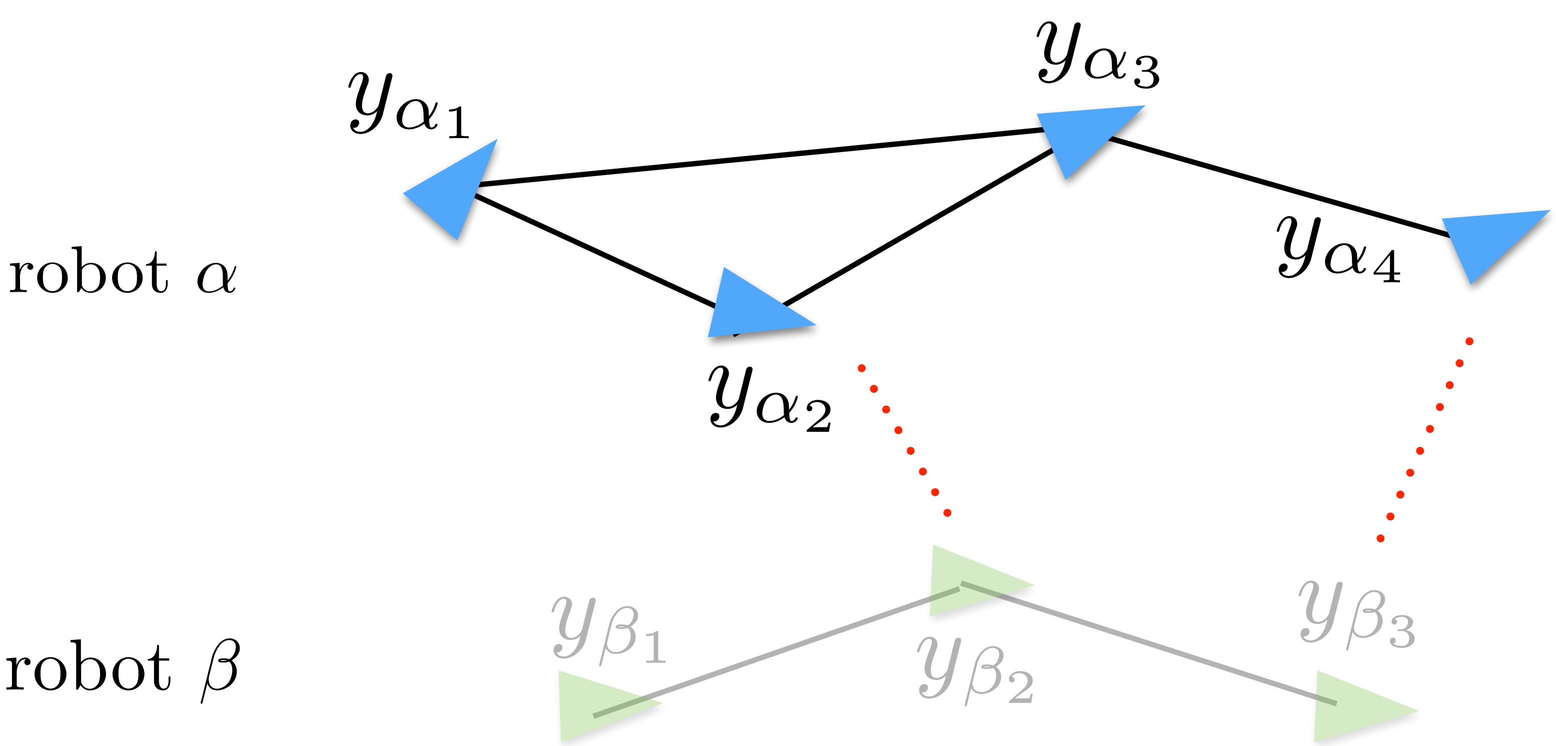

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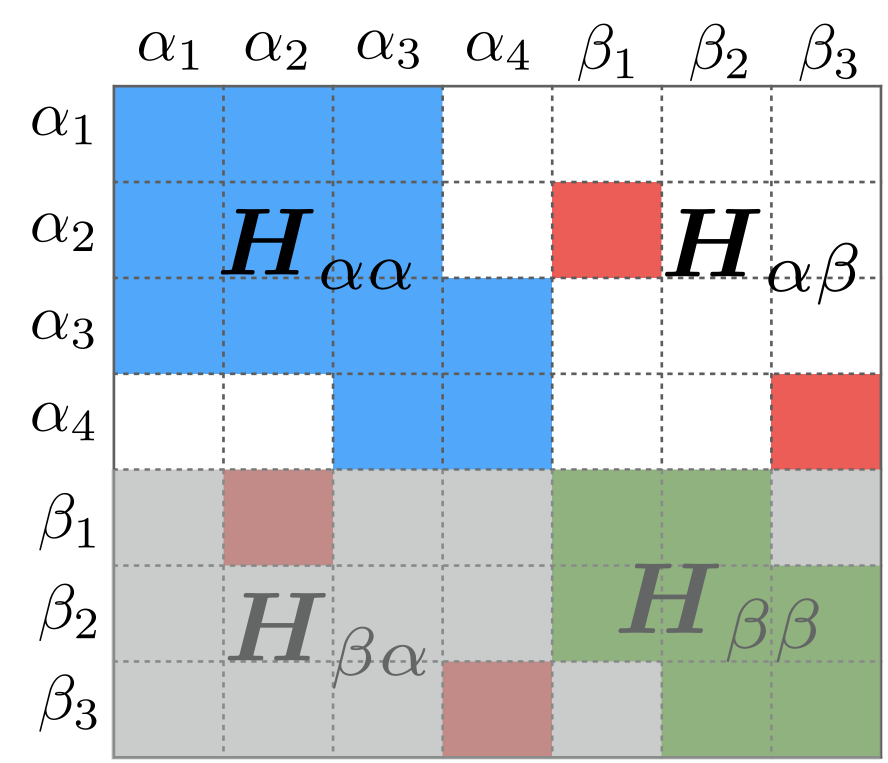

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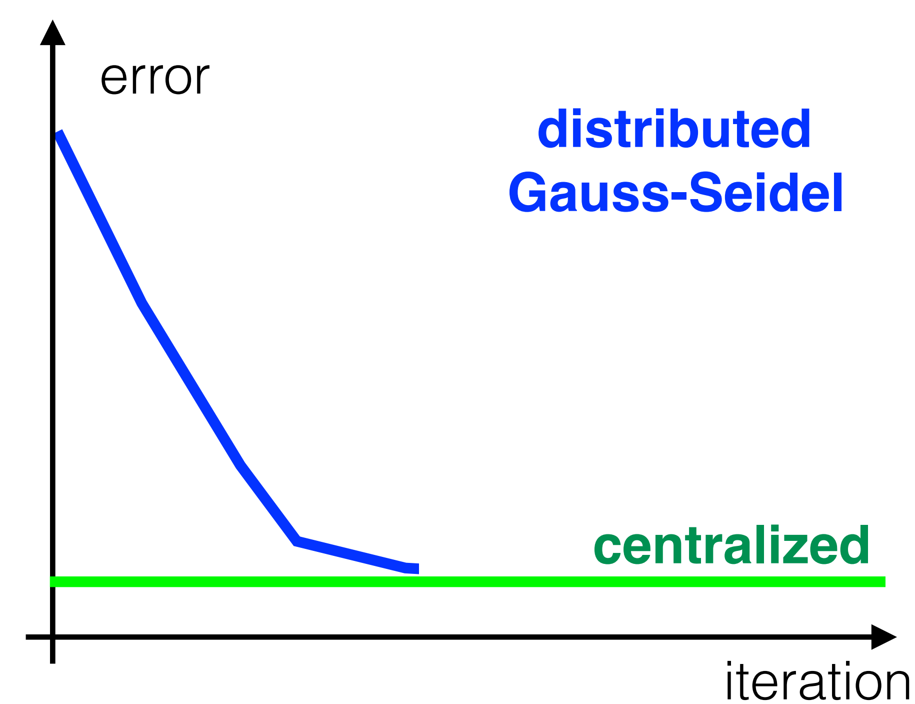

Distributed Gauss-Seidel Approach

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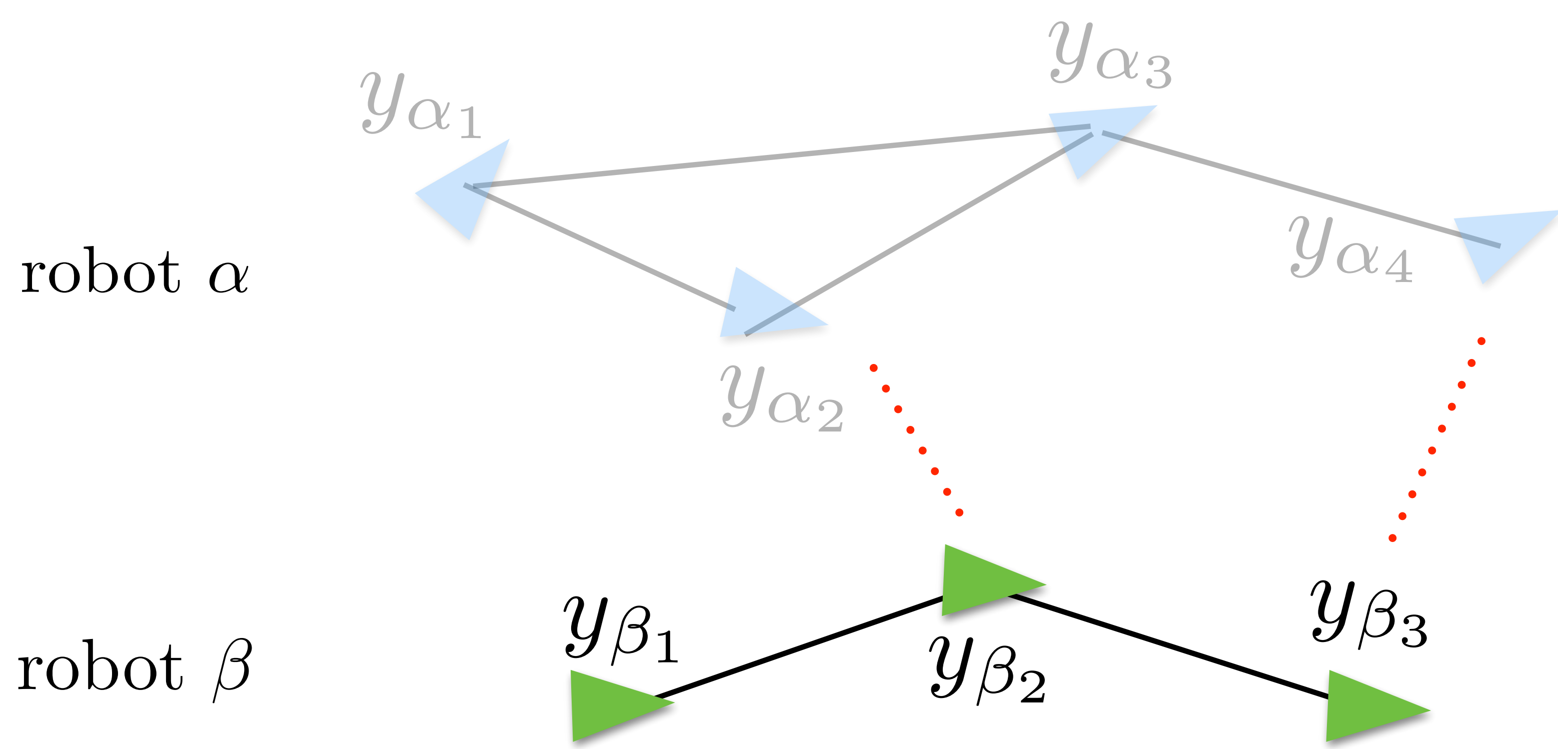

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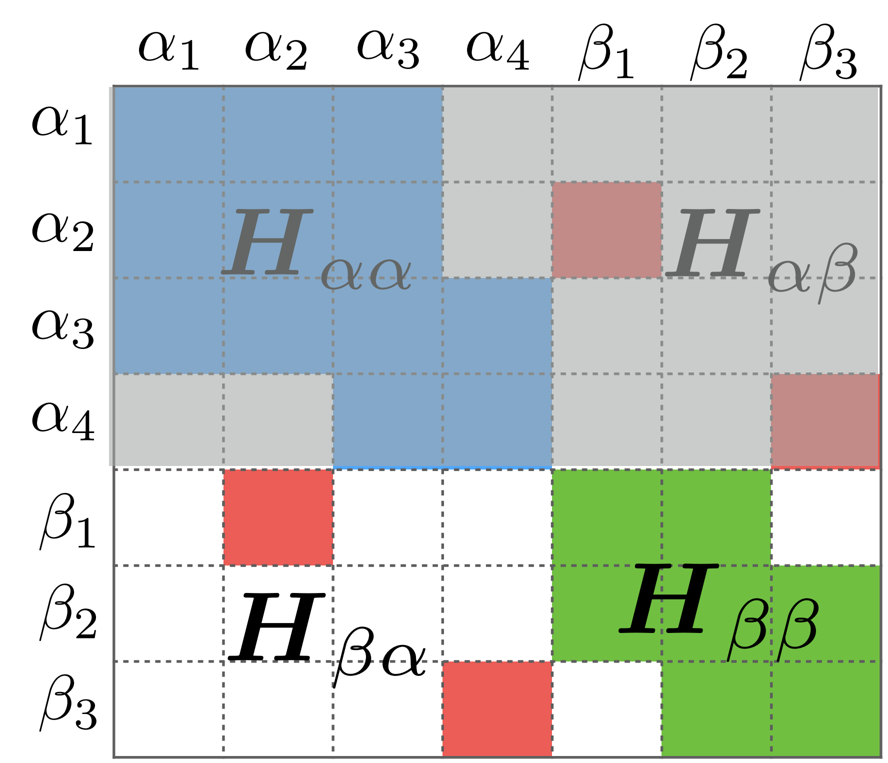

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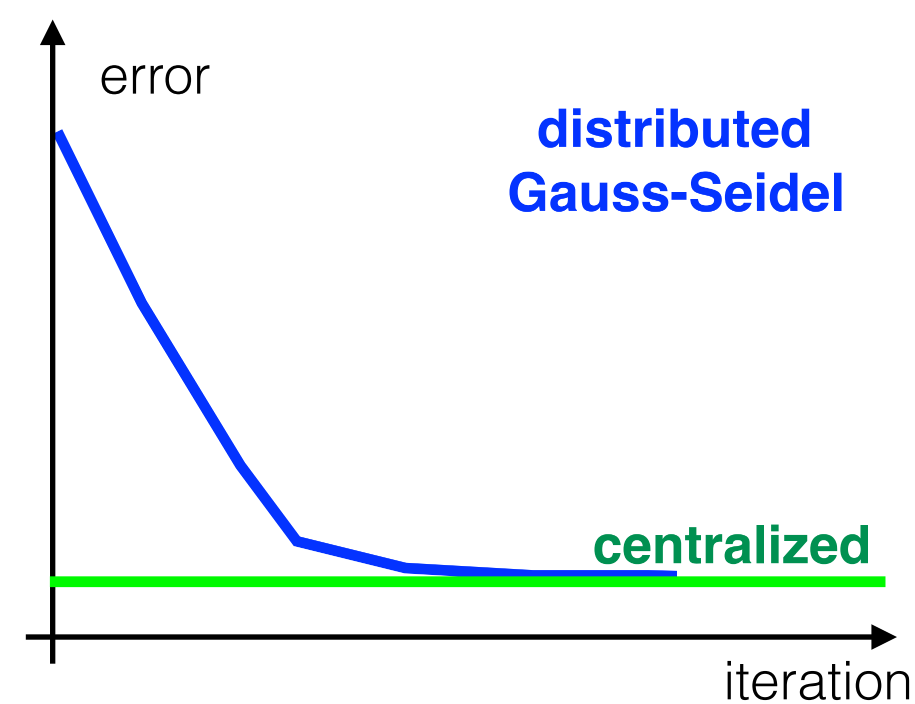
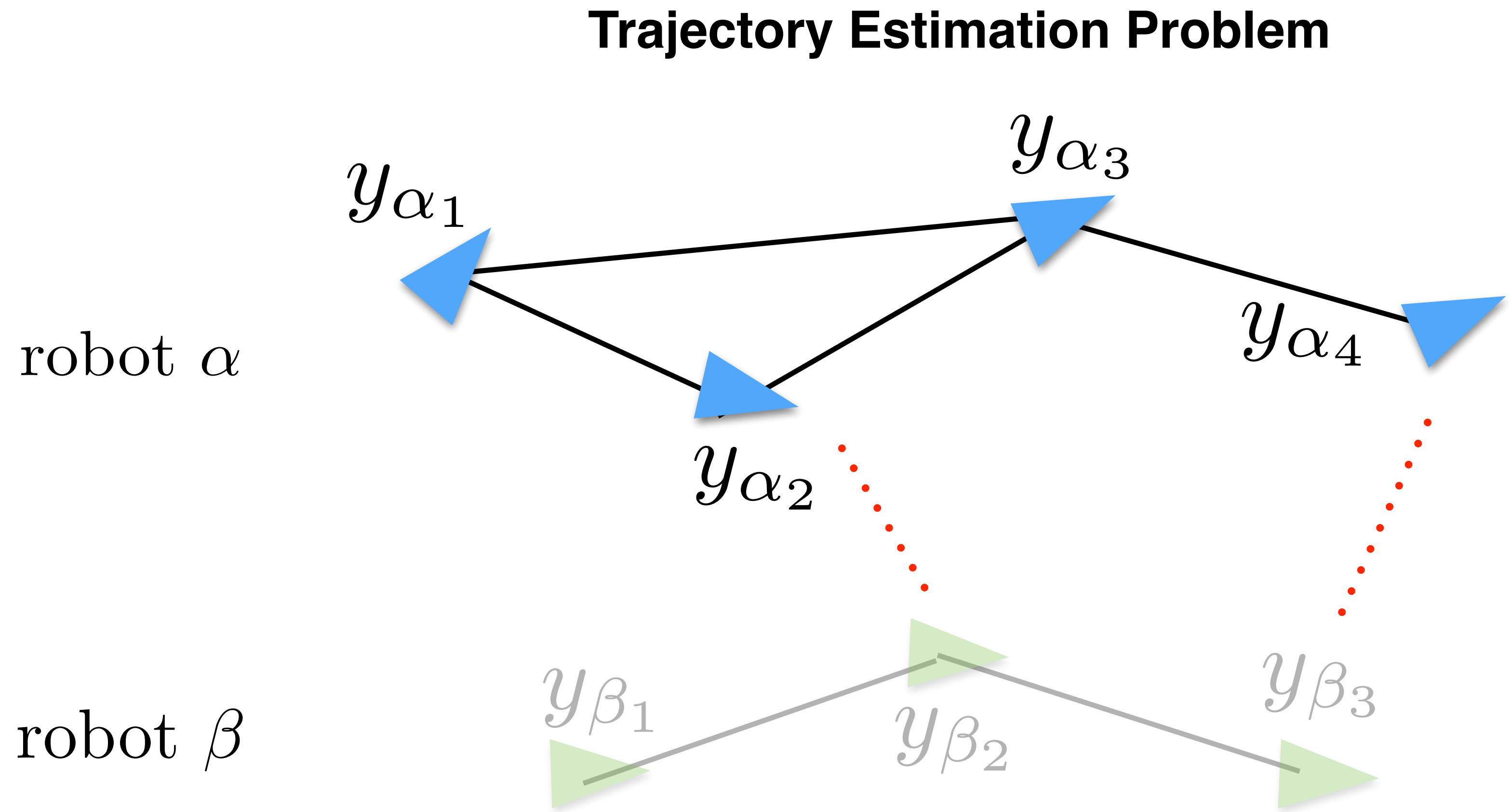
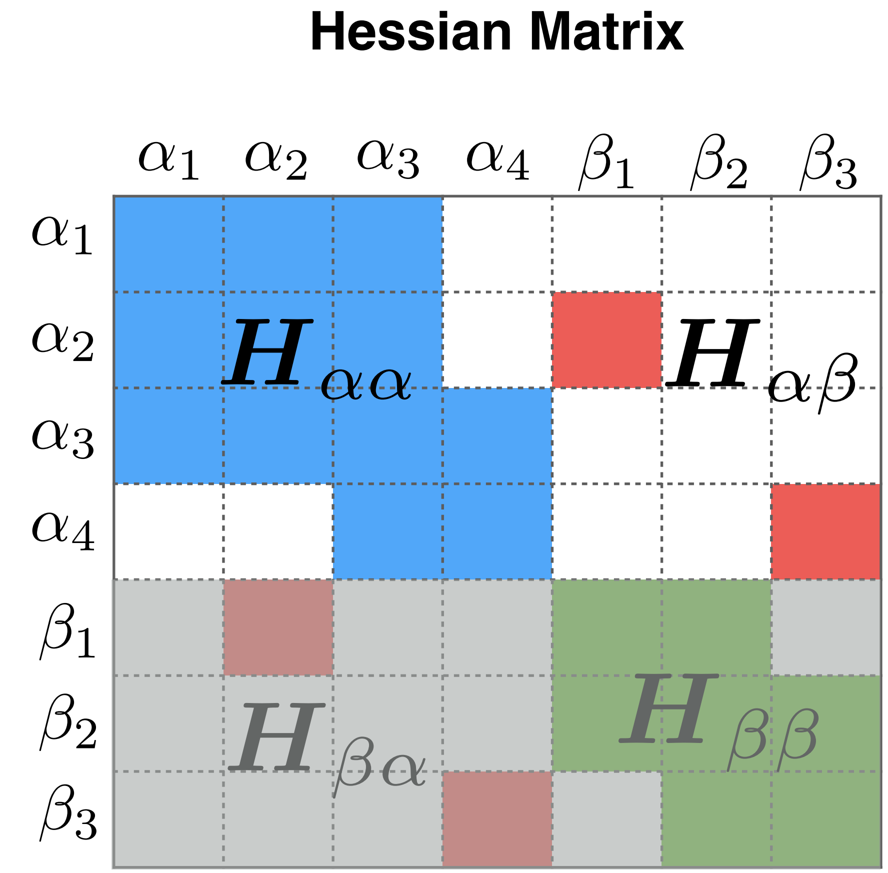


Distributed Gauss-Seidel Approach





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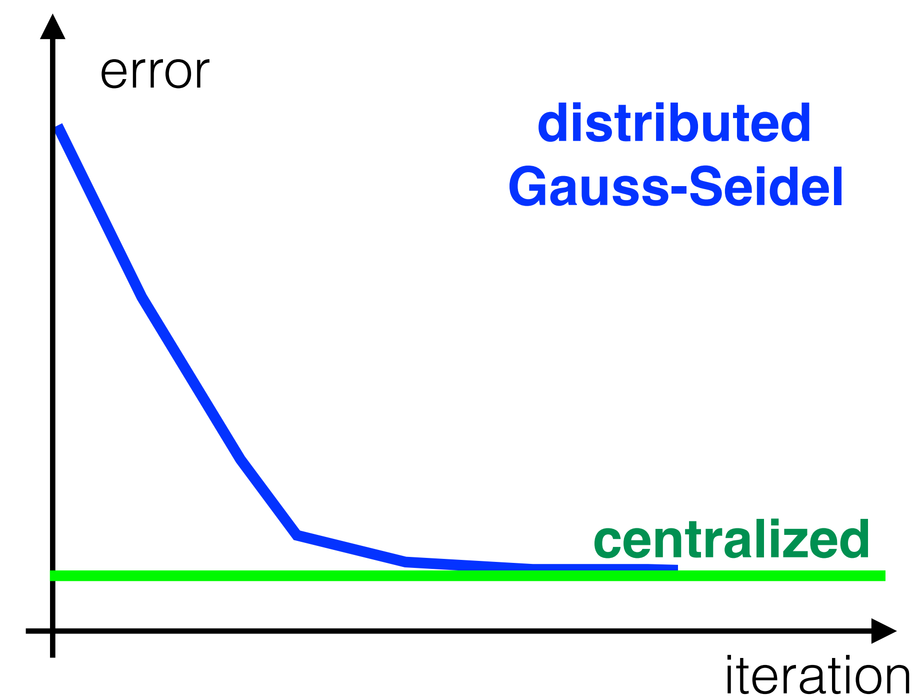
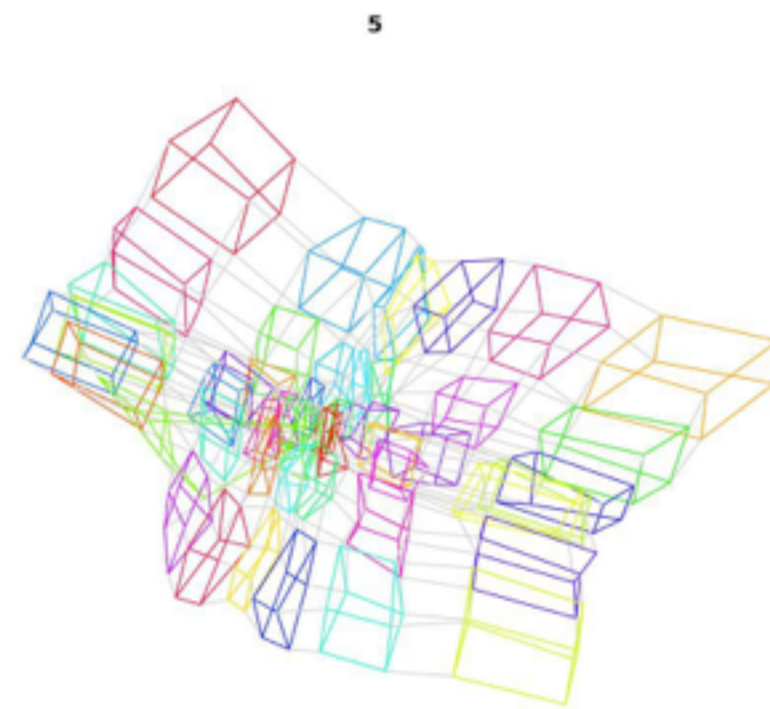


Simulation Results

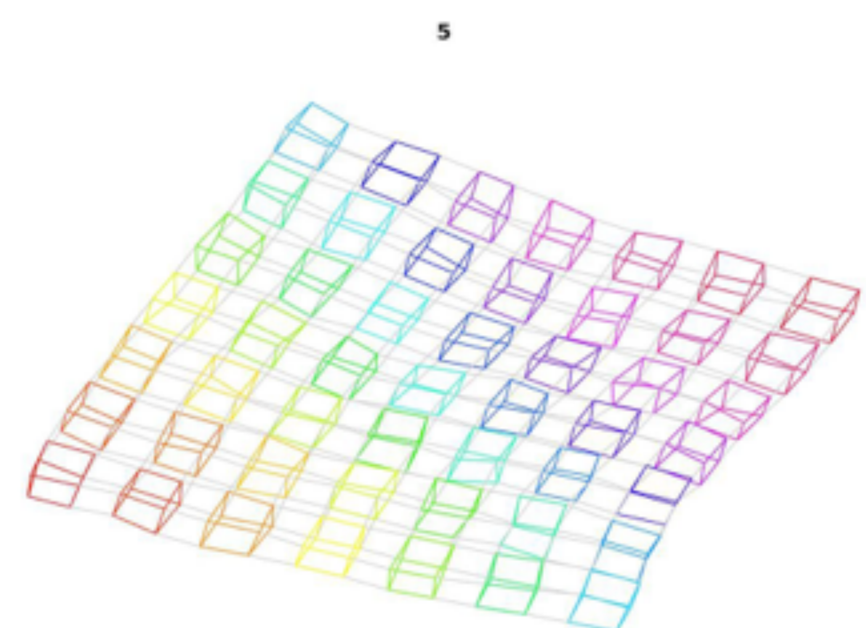
The approach has the following merits:

1. **Proven convergence to centralized. Fast convergence with smart initialization**
2. Communication is linear in number of rendezvous
3. Scalability in the number of robots
4. Resilience to noise

Without Flagged Initialization



With Flagged Initialization



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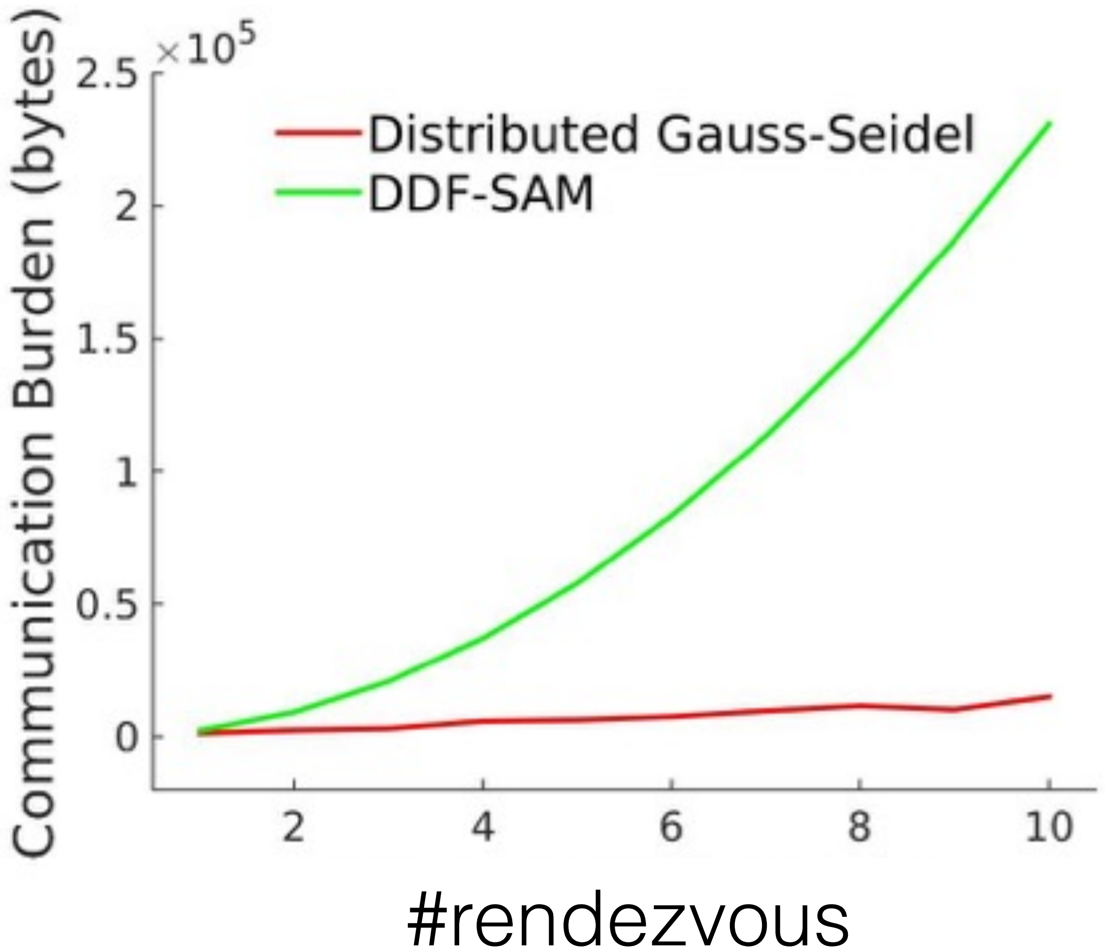

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Simulation Results

The approach has the following merits:

1. Proven convergence to centralized. Fast convergence with smart initialization

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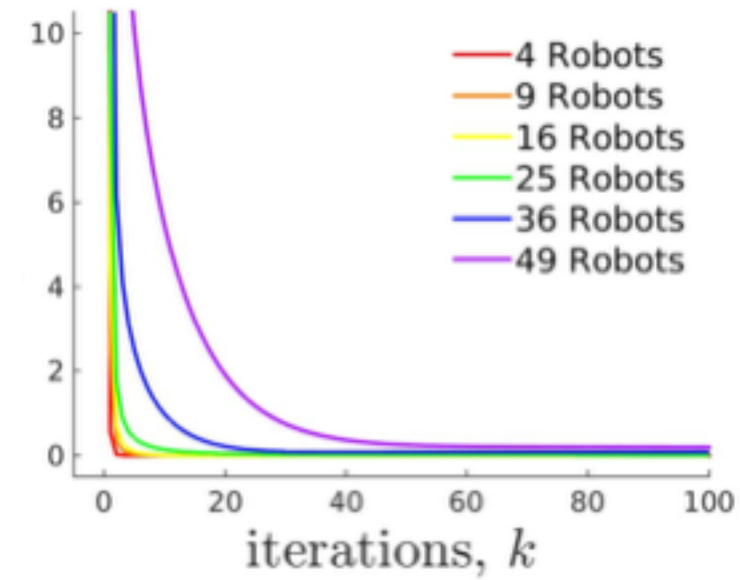
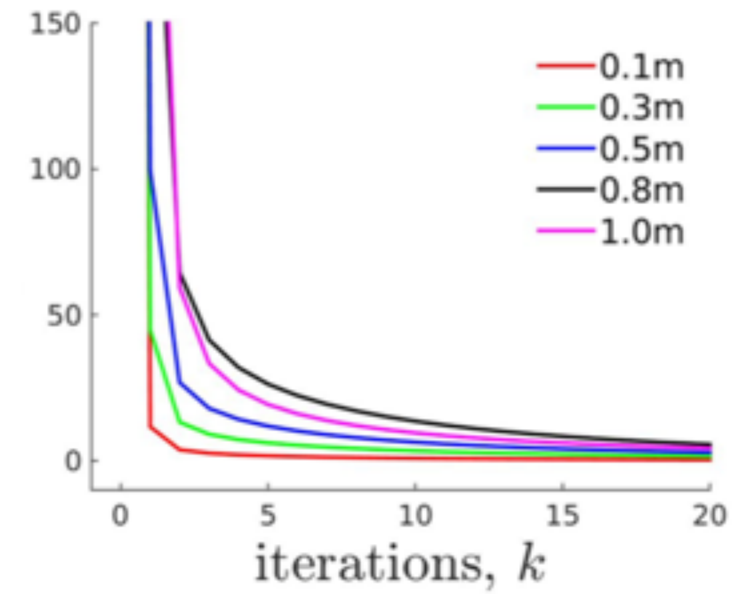
3. **Scalability in the number of robots**

4. **Resilience to noise**

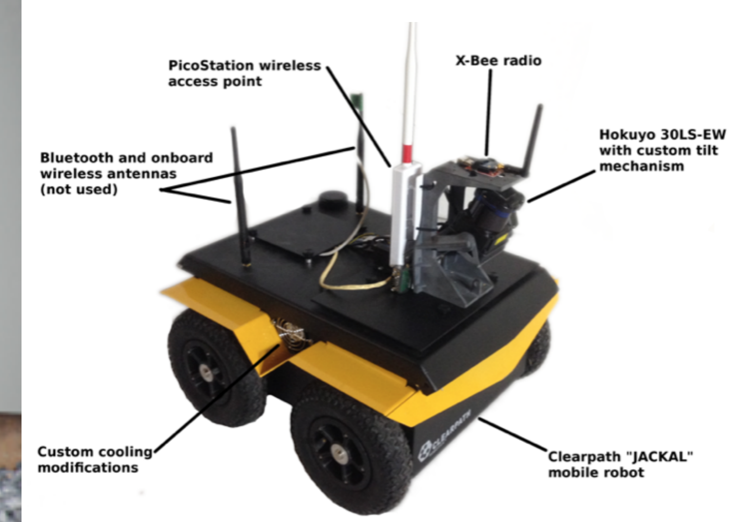
Increasing number of robots



Increasing measurement noise



Field Experiments

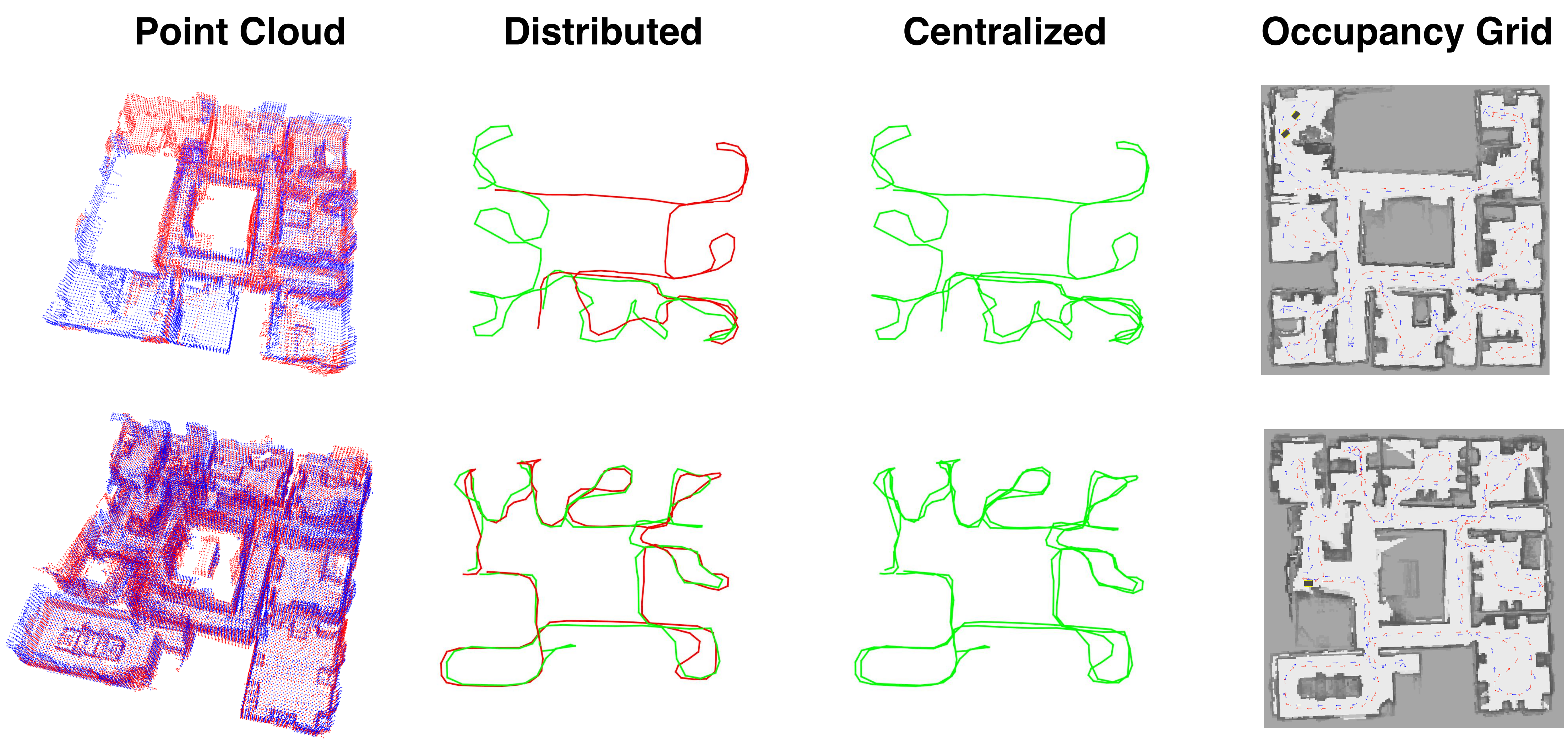


We tested the proposed approach on field data collected by two to four Jackal robots, moving in a military test facility. We use the estimated trajectories to reconstruct a 3D map of the facility.

Field Experiments (4 Robots)

Thank you!

For further information, please come to the interactive session: **1.4 (Balcony)**



Georgia Tech Robotics & Intelligent Machines

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