Simultaneous Localization and Mapping: Literature Survey

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SLAM is an active area of research in robotics. Durrant-Whyte and Bailey provide a survey of the earlier SLAM literature [9]. In this report we summarize the efforts made in the SLAM community to reduce its complexity and make it more scalable.

1 Filtering based SLAM

One of the initial solutions to the SLAM problem was proposed by Smith and Cheeseman who used the Extended Kalman Filter (EKF) to jointly represent the landmark position with the pose [38]. Guivant and Nebot [18] developed compressed EKF which performed mapping in local space using a reduced number of landmarks and only performed global update when the robot moved from one local map to another local map. The worst case complexity in this case is $\mathcal{O}(kn^2)$ required for full global update. Paskin [36] proposed thin junction tree filter (TJTF) to maintain manageable complexity. It is an assumed density filtering algorithm where the belief state is represented using a junction tree and proposed a thinning operation to maintain an ever-increasing tree width caused by filtering operations. Estimation in this case is performed in $\mathcal{O}(k^3n)$ time by passing marginalized distribution along the edges of the junction tree. According to Frese et al. [13] "From the perspective of "linear equation solving" this approach is complementary to the approach proposed in this paper. In the Gaussian case the junction tree algorithm is a direct, i.e., exact equation solver based on Schur-complements (corresponding to the marginalized distributions) and approximation is performed on the equation level. In contrast Multilevel Relaxation is an iterative, i.e., approximate equation solver but does not approximate the equations themselves."

2 Graph SLAM

Lu and Milos [29] presented the first smoothing approach and refined the map by globally optimizing a system of equations introduced by constraints. Gutmann and Konolige [19] proposed a system for incrementally solving the graph. They used matrix inversion to optimize the graph. Duckett et al. [8] presented a relaxation algorithm with better computational complexity as compared to matrix inversion. Folkesson and Christensen [12] introduced GraphSLAM system which finds the best robot trajectory using a non linear optimization technique. They used gradient descent to optimize. Dellaert and Kaess [6] exploited the inherent sparsity of the SLAM problem to make the process more efficient . These approaches solve a batch least squares optimization which become computationally expensive for large-scale problems. Incremental smoothing and mapping algorithm exploits the fact that new measurements have only a local effect on the map and therefore incrementally updates the square root information matrix with the new measurements [24]. In this approach, the length of the trajectory depends on the exploration time rather than the explored area which becomes computationally expensive for long term SLAM.

3 Graph Sparsification

Folkesson and Christensen [12] reduced the number of nodes after every 50 nodes and represented those nodes using a single marginalized star node. Eade et al. [10] reduced the complexity of the graph by marginalizing out past robot poses that are not useful in subsequent operations. This is followed by node degree based heuristic pruning of edges. It discards the constraints with the least residual which introduces bias into the system. Vial et al. [42] presented a conservative sparsification techniqhuue that minimizes the KL divergence of the information matrix for sparsely approximating multi-dimensional Gaussian distributions. However it requires inversion of information matrix which limits its utility to smaller graphs. Huang et al. [21] propose a consistent graph sparsification scheme to marginalize out old nodes while retaining all the information conveyed in the discarded measurements. However they need information matrix of the full graph in order to marginalize a single node. Wang et al. [43] performs pose graph reduction using a greedy pruning based on KL Divergence measure between the reduced graph and full graph. Carlevaris-Bianco et al. [3] proposed a generic node removal technique to produce a new set of linearized factors over the elimination cliques that represents either the true or a sparse approximation of the true marginalization. Mazuran et al. [30] formulated sparsification as a convex minimization problem where they select a set of non-linear measurements that best approximate the original distribution. It does not require a global linearization point and can be used with any non linear measurement function.

4 Landmark/Pose Reduction

Cao and Snavely [2] proposed a probilistic version of K-Cover algorithm which takes into account both point appearance and visibility for reducing SfM point clouds. Performance improvement on using probabilistic k-cover is not much as compared to the combinatorial k-cover algorithm. Park et al. [35] select a subset of 3D points using mixed-integer quadratic programming. Dissanayake et al. [7] showed that it is possible to remove a large percentage of the landmarks from the map without making the map building process statistically inconsistent. They select the least uncertain landmark among the new landmarks every time the robot traverses a fixed distance. Keyframe based approaches select a subset of keyframes and perform batch optimization only on the keyframes [25]. Kretzschmar and Stachniss proposed an information theoretic approach to compress the pose graph by selecting the most informative laser scans with respect to the map and marginalizing out poses that correspond to the discarded laser scans[26]. Ila et al. [22] proposed to add only non-redundant and informative links to the graph based on the distance between the given pair of poses and the mutual information gain when linking two poses.

Strasdat et al. [40] proposed an approach to learn landmark selection policy which allowed a robot to discard landmarks that are not valuable for its current navigation task. Monte-Carlo reinforcement learning is used to obtain the selection policy. In contrast we do not have a specific navigation goal and we try to preserve as much information as possible given the current state.

5 Distributed SLAM

Cunnigham et al. [5] extended smoothing and mapping approach to implement decentralize data fusion. They introduce constrained factor graph as a representation to perform distributed inference and optimization. One problem with this approach is that all the robots share linearized condensed graph with neighboring robots which can be sub-optimal depending on the linearization point. Batch summarization is not scalable to large local maps as well. DDF SAM 2.0 [4] made the process more efficient using anti-factor to subtract out the old summarized map and add a new summarized map. However it still cannot handle re-linearizations.

6 Submap based SLAM

Leonard and Feder [27] proposed decoupled stochastic mapping which updates only the current local submap. However the estimates can become overconfident when the robot passes from one submap to another submap. Leonard and Newman [28] propose a constant time SLAM solution which achieves near-optimal result in cases where the robot makes repeated visits to all regions of the environment. According to Leonard and Newman, “no previous method satisfies each of the three criteria of (1) provable consistency, (2) spatial convergence, and (3) constant-time updates”. But this method assumes linear Gaussian case. Map updates are performed in a local neighborhood around the current submap using the shared set of landmarks. According to Bosse et al. [1] “The hybrid metrical/topological approach allows us to restrict the representation of errors via Gaussian distributions to local regions where linearization works well, rather than representing the entire environment with one Gaussian distribution”. Frese et al. [13] proposed multi-level relaxation resulting in a linear time update. It is based on multigrid methods for solving partial differential equation which optimizes the map at multiple levels of resolution. Frese [**?**] proposed TreeMap algorithm which is similar to TJTF. It divides the environment into a parts-whole-hierarchy represented as a binary tree. Since it uses a balanced tree, update requires only $\mathcal{O}(k^3 \log n)$ time.

Estrada et al. [11] presented an hierarchical SLAM framework which consist of a set of local maps connected by arcs labelled with relative location between the maps.

As compared to previous approaches it maintains loop consistency when calculating the optimal estimate at global level. Ni et al. [32] presented a an exact submapping approach in smoothing and framework. They cache the linearization of the submaps and reuse it when they are combined into a global map. Grisetti et al. [15] updates only a part of the map that needs to be considered for performing data assocation. Everytime an observation is obtained the highest level is modified and only the areas which are substantially modified are changed at lower levels. Ni and Dellaert [33] extended their previous approach to multiple levels and used nested dissection to minimize the dependence between two subtrees. Grisetti et al. [17] proposed a robust optimizaton approach using solution of submaps to provide good initial estimate for global alignment. Condesed measurements computed from partial solutions have large convergence basin. Zhao et al. [44] present a approximation strategy for large scale SLAM by solving a sequence of submaps and joining them in a divide and conquer manner using linear least squares. Suger et al. [41] present a approximate SLAM based on hierarchical decomposition to reduce the memory consumption required to solve the complete graph. This helps in cases where the entire map does not fit into main memory.

7 Relative Parametrization

ATLAS framework [1] uses a relative framework where each vertex represents a local frame and each edge represents the transformation between adjacent frames. Dijkstra shortest path is used to project uncertainty of certain entities w.r.t arbitrary coordinate frames. The edges between adjacent submaps are represented using a linear Gaussian which introduces errors. This framework does not perform global update of inter submap edges when revisiting a previously known area. Howard et al. [20] introduced manifold maps as a self-consistent map representation for navigation. It optimizes the relative transformation between the local patches given the observations. Olson et al. [34] represent the state-space using incremental pose transformation which is then solved using stochastic gradient descent. This approach is robust to poor initial estimates. Grisetti et al. [16] improved this approach using a tree based parametrization so that the complexity is dependent on the size of the environment rather than the length of the trajectory.

Sibley et al. [37] proposed relative formulation and bundle adjustment for optimizing the graph instead of optimizing it using a single privileged frame. This helps in loop closure when the error in relative formulation propogates only to some nearby edges and does not propagate around the entire loop as in the case of absolute formulation. The network of relative parameters are critically damped. Mei et al. [31] presented a stero SLAM system that used continuous relative representation and showed that it is possible to represent trajectories in continous relative representation that cannot be embedded in a Euclidean space. An example Mei et al. [31] give is “An indoor sequence was made with the exploration of the second floor of a building followed by a flight of stairs to the first floor. The elevator was then taken back up to the second floor. A loop closure was triggered on the second floor that created an edge transform in the graph. The reprojection error does not increase in this framework. However the trajectory cannot be represented in Euclidean space so a global bundle adjustment for example would fail. A BFS used to draw the map shows a tear. Travelling in the lift has removed the compatibility between a relative and a Euclidean representation. The transition between the floor building and the lift is the transition between two maps that can be represented in a Euclidean space. It is important to note that the relative map stays usable for navigation and path planning." Strasdat et al. [39] presented a double window optimization framework for constant time visual SLAM. According to them, "RBA is equivalent to BA if the network of relative constraints forms a tree. Thus, it works especially well on exploratory scenarios where there are no cycles within the active window. However, the accuracy of RBA degrades if there are many loops, as it does not enforce the condition that constraints around the loop add up to identity."

8 Long Term SLAM

Johannsson et al. [23] used a reduced pose graph representation to bound the size of the pose graph with respect to the explored area, instead of exploration time. The new constraints are added between existing nodes instead of adding new nodes to the graph when the robot revisits the same location. Visual teach and repeat [14] is another important direction for long term autonomy. They build a manifold map of overlapping submaps during learning phase which can be repeated autonomously for long routes.

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