Information-based Reduced Landmark SLAM

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Abstract— In this paper, we present an information-based approach to select a reduced number of landmarks and poses for a robot to localize itself and simultaneously build an accurate map. We develop an information theoretic algorithm to efficiently reduce the number of landmarks and poses in a SLAM estimate without compromising the accuracy of the estimated trajectory. We also propose an incremental version of the reduction algorithm which can be used in SLAM framework resulting in information based reduced landmark SLAM. The results of reduced landmark based SLAM algorithm are shown on Victoria park dataset and a Synthetic dataset and are compared with standard graph SLAM (SAM [6]) algorithm. We demonstrate a reduction of 40-50% in the number of landmarks and around 55% in the number of poses with minimal estimation error as compared to standard SLAM algorithm.

I. Introduction

Simultaneous localization and mapping (SLAM) is one of the basic problems in mobile robotics. SLAM allows the robot to incrementally build a consistent map of the environment while simultaneously using the map to localize itself. It has a wide range of applications from service robotics to reconnaissance operations. The complexity of the existing SLAM approaches grow with the length of the robot's trajectory which challenges long term autonomy. Some recent approaches solve this problem by compressing the pose graph to reduce its complexity [9], [17], [14], [12], [2], [23], [18]. These methods sparsify the pose graph to reduce the number of poses and do not consider the size of the generated map.

As compared to pose graph based SLAM, landmark based SLAM explicitly maintains the landmark and robot states. Keeping landmarks in the SLAM estimate has advantages over marginalizing out. For example, semantic landmarks like planes and objects can be used to perform tasks which require higher cognition capability. Additionally, it can be associated with the current robot pose and therefore be used to localize the robot [24], [4]. However storing all landmarks and poses can become expensive in the long term.

Therefore, in this paper we address this problem by developing an information-based approach to select a reduced number of landmarks and poses for a robot to successfully localize itself and simultaneously build an accurate map. We present an incremental and active minimization approach

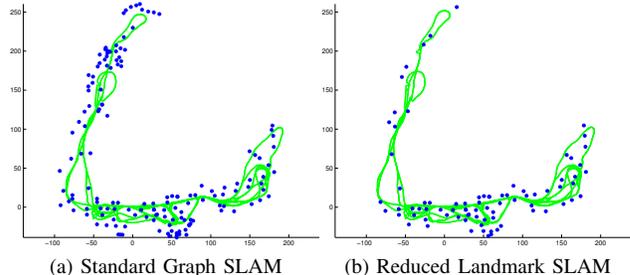(a) Standard Graph SLAM   (b) Reduced Landmark SLAM

Fig. 1: Comparison of pose trajectory and landmark locations estimated using (a) Standard Graph SLAM (SAM [6]) and (b) Reduced Landmark based SLAM on Victoria Park dataset.

(Section IV) for doing reduced landmark based SLAM. The active minimization approach uses an information theoretic algorithm (Algorithm 3) to reduce the number of landmarks and poses. Performing the minimization in an incremental manner (Algorithm 5) results in a reduced landmark based SLAM algorithm. We evaluate our approach using two datasets, Victoria Park [11] and a synthetic dataset comparing it to standard graph SLAM (SAM [6]) and demonstrating a reduction of 40-50% in the number of landmarks and around 55% in the number of poses with minimal estimation error. Figure 1 shows a comparison of robot trajectory and landmark locations estimated using standard graph SLAM and reduced landmark based SLAM on Victoria Park dataset.

Our contributions in this paper are as follows:

1) We develop an information theoretic algorithm to efficiently reduce the number of landmarks and poses without compromising the accuracy of the estimated trajectory.
2) We propose an incremental version of this algorithm which can be used in a SLAM framework required for online operations.

In Section II, we discuss the related work in this area. Section III reviews landmark based SLAM and proposes a formulation for reduced landmark based SLAM. Section IV describes the incremental and active minimization algorithms for doing the same. In Section V we give an extensive evaluation of our approach on two datasets and we conclude in Section VI.

II. Related Work

SLAM is an active area of research in robotics. One of the initial solutions to the SLAM problem was proposed by Smith and Cheeseman who used the Extended Kalman Filter (EKF) to jointly represent the landmark position with the

pose [21]. Durrant-Whyte and Bailey provide a survey of the SLAM literature [8].

Dissanayake et al. [7] showed that it is possible to remove a large percentage of the landmarks from the map without making the map building process statistically inconsistent. Folkesson and Christensen introduced the GraphSLAM system which finds the best robot trajectory using a non linear optimization technique [10]. Dellaert and Kaess exploited the inherent sparsity of the SLAM problem to make the process more efficient [6]. These approaches solve a batch least squares optimization which become computationally expensive for large-scale problems.

Various other methods have been developed to reduce the computational time. Sibley et al. used sliding window filter whereas Ni et al. divided the whole graph into multiple local sub-graphs [19], [20]. Incremental smoothing and mapping algorithm updates the square root information matrix with the new measurements [16]. In this approach, the length of the trajectory depends on the exploration time rather than the explored area which becomes computationally expensive for long term SLAM.

Kretzschmar and Stachniss proposed an information theoretic approach to compress the pose graph by selecting the most informative laser scans with respect to the map [17]. Ila et al. [13] proposed to add only non-redundant and informative links to the graph. Eade et al. [9] reduced the complexity of the graph by marginalizing out past robot poses that are not useful in subsequent operations. Vial et al. [25] presented a conservative sparsification technique that minimizes the KL divergence of the information matrix for sparsely approximating multi-dimensional Gaussian distributions. Huang et al. [12] propose a consistent graph sparsification scheme to marginalize out old nodes. Wang et al. [26] performs pose graph reduction using a greedy pruning based on KL Divergence measure between the reduced graph and full graph. Johannsson et al. [14] used a reduced pose graph representation to bound the size of the pose graph with respect to the explored area. Carlevaris-Bianco et al. [2] proposed a generic node removal technique to produce a new set of linearized factors over the elimination cliques that represents either the true or a sparse approximation of the true marginalization. Mazuran et al. [18] formulated sparsification as a convex minimization problem. All of these compress the pose graph by reducing views or poses to reduce the complexity of graph optimization where as our approach reduces the number of landmarks and poses in a landmark based SLAM framework.

Suger et al. [23] present an approximate SLAM based on hierarchical decomposition to reduce the memory consumption. Cao and Snavely [1] proposed a probabilistic K-cover algorithm to reduce the size of 3D reconstructions. Chli et al. used mutual information between the measurements to guide the search during feature matching step of a visual SLAM system [3]. Kaess and Dellaert ignore the feature measurements whose information gain falls below a specified threshold [15]. In contrast we estimate the information provided by a landmark rather than a measurement.

III. APPROACH

We develop an incremental and active minimization approach (Section IV) for doing reduced landmark-based SLAM. While the robot is navigating, reduced landmark based SLAM selects a reduced number of landmarks and poses from the environment for a robot to localize itself and simultaneously build an accurate map. The active minimization approach uses an information theoretic algorithm (Algorithm 3) to reduce the number of landmarks and poses. Incremental minimization performs the same in an iterative manner (Algorithm 5).

In the following sections we review the standard graph SLAM formulation (using all landmarks and poses) and propose a formulation for reduced landmark-based SLAM.

A. Landmark-based SLAM

In landmark-based SLAM a robot while navigating tries to localize itself and at the same time build a map of the environment (represented using landmarks). Assuming the pose of the robot at i^{th} time step is x_i with $i \in 0 \ldots M$, a landmark is l_j with $j \in 1 \ldots N$ and a measurement is z_k, with $k \in 1 \ldots K$, the joint probability model is given as,

$$P(X, L, Z) = P(x_o) \prod_{i=1}^{M} P(x_i|x_{i-1}, u_i) \prod_{k=1}^{K} P(z_k|x_{ik}, l_{jk})$$

where $P(x_o)$ is a prior on the initial state, $P(x_i|x_{i-1}, u_i)$ is the motion model, parameterized by a control input or odometry measurement u_i and $P(z_k|x_{ik}, l_{jk})$ is the landmark measurement model where x_{ik} (i^{th} pose) and l_{jk} (j^{th} landmark) correspond to the z_k measurement. X is the set of poses, L is the set of landmarks and Z is the set of measurements. Assuming the motion and measurement models are Gaussian, $P(x_i|x_{i-1}, u_i) \propto \exp{-\frac{1}{2}\|f_i(x_{i-1}, u_i) - x_i\|^2_{\Lambda_i}}$ and $P(z_k|x_{ik}, l_{jk}) \propto \exp{-\frac{1}{2}\|h_k(x_{ik}, l_{jk}) - z_k\|^2_{\Sigma_k}}$ where $f()$ is the robot motion equation and $h()$ is a landmark measurement equation with Λ_i and Σ_k as the respective covariances.
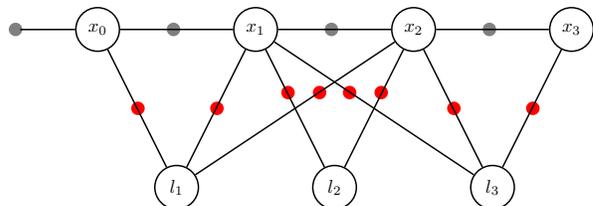

Fig. 2: Factor Graph corresponding to the Landmark based SLAM problem. The pose of the robot at i^{th} time step is x_i with $i \in 0 \ldots M$ and a landmark is l_j with $j \in 1 \ldots N$. Measurements z_k with $k \in 1 \ldots K$ are shown by small filled circles corresponding to a factor node.

We use a factor graph to represent the joint probability model $P(X, L, Z)$ where each factor represents either $P(x_o)$

or $P(x_i|x_{i-1}, u_i)$ or $P(z_k|x_{ik}, l_{jk})$. Therefore the joint probability model can be written as

$$P(X, L, Z) \propto g(\Theta) = \prod_i g_i(\Theta_i) \qquad (1)$$

where Θ_i is the set of variables θ_j adjacent to the factor g_i. Figure 2 shows the corresponding factor graph. Given all the measurements, we obtain the maximum a posteriori (MAP) estimate Θ^* by maximizing the joint probability $P(X, L, Z)$.

$$\Theta^* = \arg \max_{\Theta} P(X, L|Z) = \arg \min_{\Theta} (-\log g(\Theta)) \quad (2)$$

which leads to the following non-linear least squares problem:

$$\Theta^* = \arg \min_{\Theta} \sum_{i=1}^{M} \|f_i(x_{i-1}, u_i) - x_i\|_{\Lambda_i}^2 \qquad (3)$$
$$+ \sum_{k=1}^{K} \|h_k(x_{ik}, l_{jk}) - z_k\|_{\Sigma_k}^2$$

The non-linear least squares problem in Equation 3 is solved using non-linear optimization method such as Levenberg-Marquardt algorithm which solves a succession of linear approximations to approach the minimum.

In each iteration of the non-linear least squares problem, we linearize around a linearization point Θ to obtain a linear least squares problem of the form $\delta^* = \arg \min_{\delta} \|A\delta - b\|_2^2$ where A represents the Jacobian matrix, b is the set of measurements (odometry or landmark) and δ represents the change around the current estimate of X and L. For a full rank matrix A, the least squares solution can be found by solving the normal equations $A^T A \delta^* = A^T b$. Cholesky factorization yields $A^T A = R^T R$ where R is an upper-triangular matrix. A forward substitution on $R^T y = A^T b$ followed by backward substitution $R\delta^* = b$ gives the update δ^*. We use the Georgia Tech Smoothing and Mapping (GTSAM) library to jointly optimize the robot poses and landmarks [5].

B. Reduced Landmark-based SLAM

The goal of reduced landmark-based SLAM is to navigate in an unknown environment using the reduced amount of landmarks and poses. It can be formulated as incrementally finding a reduced subset of landmarks and robot poses (L_s, X_s) such that the difference between trajectory estimated using a reduced subset of landmarks and robot poses and the trajectory estimated using all landmarks and robot poses is minimal.

However, there is a trade-off between the memory requirement and the estimation error because reducing the number of landmarks (memory requirement), increases the error in the estimated trajectory. To represent the trade-off between the memory requirements and the estimation accuracy, we formulate the problem as finding the subset of landmarks $L_s \in L$ and robot poses $X_s \in X$ which minimizes the objective function given as,

$$\rho(L_s, X_s) = (1 - \lambda)d(\Theta_s^*) + \lambda m(L_s, X_s) \qquad (4)$$

where $d()$ is the distance function representing the accuracy of the trajectory estimated (Θ_s^*) using the subset of landmarks L_s and poses X_s, $m()$ is the amount of memory required to store the same and λ is the weight parameter. The objective function represents the trade-off using λ as the weight parameter. Higher λ will force the optimization to remove as many landmarks and poses as possible whereas lower λ will be more conservative by retaining more landmarks and poses. $d()$ and $m()$ are normalized to have values between 0 and 1. Equation 4 is discussed in Section IV-B.

The reduced set of landmarks and poses is estimated by minimizing Equation 5.

$$\{L^*, X^*\} = \arg \min_{L_s, X_s} \rho(L_s, X_s) \qquad (5)$$

Directly minimizing the above objective function is infeasible since we have to iterate over all combinations of landmarks and poses which is of the order 2^N where N is total number of poses and landmarks. Below we describe an incremental and active minimization approach for the same.

IV. REDUCED LANDMARK BASED SLAM VIA INCREMENTAL AND ACTIVE MINIMIZATION

We estimate the subset of landmarks and poses by actively removing the least informative landmark until the objective function (Equation 4) has its minimum value. The corresponding poses that do not see any landmark are marginalized out. Below we describe information gain and the objective function used followed by the description of active and incremental minimization algorithms.

A. Information Gain

Mutual information $I(\alpha, \beta)$ is the measure of the expected information gain in α on measuring the exact value of β or vice versa. The mutual information of two continuous multivariate PDFs $p(\alpha)$ and $p(\beta)$ is

$$I(\alpha; \beta) = H(\alpha) - H(\alpha|\beta) = \mathbb{E}\left[\log_2 \frac{p(\alpha|\beta)}{p(\alpha)}\right]$$

In the case of multivariate Gaussian describing a state vector $\mathbf{x}$ by mean vector μ and covariance matrix Σ such that

$$\mathbf{x} = \begin{pmatrix} x_\alpha \\ x_\beta \end{pmatrix}, \mu = \begin{pmatrix} \mu_\alpha \\ \mu_\beta \end{pmatrix}, \Sigma = \begin{bmatrix} \Sigma_{\alpha\alpha} & \Sigma_{\alpha\beta} \\ \Sigma_{\beta\alpha} & \Sigma_{\beta\beta} \end{bmatrix}$$

where x_α and x_β are the two disjoint partitions of the state vector x. The mutual information between the two partitions x_α and x_β is given by

$$I(\alpha; \beta) = \frac{1}{2} \log_2 \frac{|\Sigma_{\alpha\alpha}|}{|\Sigma_{\alpha|\beta}|} \qquad (6)$$

where $\Sigma_{\alpha|\beta}$ is the covariance matrix corresponding to $p(\alpha|\beta)$ and $\Sigma_{\alpha\alpha}$ is the covariance matrix corresponding to $p(\alpha)$. The computation of log covariance determinant $\log_2 |\Sigma|$ can be expensive for a large state space $\mathbf{x}$. Since SAM [6] maintains the square root information matrix R we
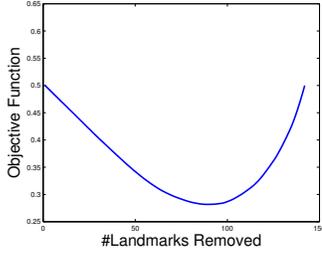
Fig. 3: Objective function for Victoria park dataset using $\lambda = 0.5$ having 6536 poses and 149 landmarks.

can compute the log of covariance determinant using the determinant of R matrix as shown below.

$$\log_2 |\Sigma| = \log_2 |A^T A|^{-1} = \log_2 |R^T R|^{-1} = -2 \log_2 |R| \tag{7}$$

R is an upper triangular matrix and therefore, the determinant of R is the product of its diagonal terms. When R matrix is not available we have to re-linearize the factor graph $g(\Theta)$ to get A followed by QR factorization to get R. Algorithm 1 shows the overall process.

Algorithm 1 LogDet $(X, g(\Theta), \Theta^*)$

1: Linearize $g(\Theta)$ around Θ^* to get A
2: Eliminate X from A to get R (QR factorization)
3: $logDet = -2 \times \sum_i (\log_2 R_{ii})$ ▷ Equation 7
4: return $logDet$

Algorithm 2 InfoGain $(X, L, g(\Theta), \Theta^*)$

1: $X_r \leftarrow \{X/L\}$
2: $logDet_\Theta \leftarrow \text{LogDet}(X_r, g(\Theta), \Theta^*)$ ▷ $\log_2 |\Sigma_{\mathbf{x}|L}|$
3: $g_r(\Theta) \leftarrow$ Remove factors having L from $g(\Theta)$
4: $\Theta_r^* \leftarrow \{\Theta_s^*/\Theta_L^*\}$
5: $logDet_{\Theta_r} \leftarrow \text{LogDet}(X_r, g_r(\Theta), \Theta_r^*)$ ▷ $\log_2 |\Sigma_{\mathbf{xx}}|$
6: return $\frac{1}{2} \times (logDet_{\Theta_r} - logDet_\Theta)$ ▷ $\frac{1}{2} \log_2 \frac{|\Sigma_{\mathbf{xx}}|}{|\Sigma_{\mathbf{x}|L}|}$

B. Objective Function

We now discuss in detail the components of the objective function $\rho(L_s, X_s)$.

Distance Function $d(\Theta_s^*)$: The distance function represents the accuracy of the trajectory Θ_s^* estimated using a reduced set of poses X_s and landmarks L_s. For a consistent estimator, the accuracy of the estimated trajectory is proportional to the uncertainty in Θ_s^*. As shown in Equation 6, information gain represents the expected reduction in uncertainty on measuring a landmark and analogously it represents the expected increase in uncertainty on removing a landmark.

Therefore we use information gain of the least informative landmark as the distance function corresponding to uncertainty in the estimated trajectory. Information gain of the least informative landmark provides a lower bound on the information gain estimated using any of the remaining landmarks. It also represents the minimum increase in uncertainty

on removing a landmark. This distance function works good in practice, although better distance function is an interesting area for future work.

Memory Consumption $m(L_s, X_s)$: As we remove the least informative landmark during each iteration and estimate the value of the objective function, we use the number of landmarks remaining as the value of $m(L_s, X_s)$. Figure 3 shows the normalized objective function for Victoria park dataset using $\lambda = 0.5$ having 6536 poses and 149 landmarks.

C. Active Minimization

Using the above formulations, we propose an active, batch minimization algorithm as summarized in Algorithm 3. The input to the algorithm is the current state of SLAM described by the set of landmarks L, robot poses X, measurements Z, the optimal landmark location and trajectory estimate Θ^* obtained by maximizing the probability $P(X, L, Z) \propto g(\Theta)$ and the weight vector λ. The value of λ is set according to how conservative the minimization has to be. By default, we use $\lambda = 0.5$ to get a good balance between the memory function value and the distance function value.

Algorithm 3 Active Minimization $(X, L, \Theta^*, g(\Theta), \lambda)$

1: Initialize $L_s \leftarrow L$, $X_s \leftarrow X$, $\Theta_s^* \leftarrow \Theta$
2: Initialize $\rho(L_s, X_s) \leftarrow \lambda m(L_s, X_s) = \lambda |L_s|$
3: **repeat**
4:    $(L_o, IG) \leftarrow$ **LandmarkSelection** $(X_s, L_s, g(\Theta), \Theta^*)$
5:    $X_r \leftarrow X_s, L_r \leftarrow \{L_s/L_o\}$ ▷ Remove L_o from L_s
6:    $\Theta_r^* \leftarrow \{\Theta_s^*/\Theta_{L_o}^*\}$ ▷ Remove $\Theta_{L_o}^*$ from Θ_s^*
7:    $d(\Theta_r^*) \leftarrow IG$, $m(L_r, X_r) \leftarrow |L_r|$
8:    $\rho(L_r, X_r) \leftarrow (1 - \lambda)d(\Theta_r^*) + \lambda m(L_r, X_r)$ ▷ Eq: 4
9:    **if** $\rho(L_r, X_r) > \rho(L_s, X_s)$ **then**
10:      break   ▷ Minimum value
11:    **end if**
12:    $L_s \leftarrow L_r$, $X_s \leftarrow X_r$, $\Theta_s^* \leftarrow \Theta_r^*$
13:    $\rho(L_s, X_s) \leftarrow \rho(L_r, X_r)$
14: **until** it breaks out of the loop
15: $X_r \leftarrow$ Poses not seeing any landmark $l \in L_s$
16: $X_s \leftarrow \{X_s/X_r\}$ ▷ Marginalize out X_r
17: return $\{X_s, L_s\}$

Algorithm 4 LandmarkSelection $(X, L, g(\Theta), \Theta^*)$

1: $minIG \leftarrow \infty$
2: $selectedLandmark \leftarrow \emptyset$
3: **for all** landmarks L_i in L **do**
4:    $IG(\mathbf{X}; L_i) \leftarrow$ **InfoGain**(X, L_i, $g(\Theta)$, Θ^*)
5:    **if** $IG(\mathbf{X}; L_i) < minIG$ **then**
6:      $minIG \leftarrow IG(\mathbf{X}; L_i)$
7:      $selectedLandmark \leftarrow L_i$
8:    **end if**
9: **end for**
10: return $(selectedLandmark, minIG)$

During each iteration of the active minimization algorithm the least informative landmark is removed until Equation 4

has reached its minimum value. A landmark is removed by deleting the landmark and the corresponding factors from the graph. The least informative landmark is selected using Algorithm 4. It evaluates the mutual information between each landmark and rest of the states by computing the difference between log of covariance determinants before and after removing the landmark (Algorithm 2). Additional results comparing various landmark selection algorithms can be found at `http://www.cc.gatech.edu/~choudhar/icra15/supplementary.pdf`.

When Equation 4 has reached its minimum value, robot poses that do not see those landmarks are marginalized out. The optimal trajectory and landmark location is re-estimated using the reduced set of landmarks and poses. This is a batch algorithm.

Landmarks that are currently being measured have low information gain and can be removed by the active minimization algorithm. However these landmarks can become important in the future. To avoid throwing away the landmarks that are being measured right now, we do not consider landmarks seen by the last P poses during the active minimization process. We use $P = 5$ during the experiments to ensure a reasonable pose lag.

The current algorithm requires all the landmarks and poses to be available before running the minimization as a batch process. Below we give an incremental version of this algorithm.

D. Incremental Minimization

To be used in a SLAM framework, landmark and pose minimization has to be done in an incremental manner. We propose an incremental minimization algorithm where active minimization (Algorithm 3) is performed after every Ψ poses of running SLAM. The reduced set of poses and landmarks are used from that point forward instead of using all landmarks and poses. The incremental minimization algorithm is shown in Algorithm 5.

Algorithm 5 Incremental Minimization(Ψ, λ)

1: Initialize $L \leftarrow \emptyset$, $X \leftarrow \emptyset$, $\Theta^* \leftarrow \emptyset$, $g(\Theta) \leftarrow \emptyset$
2: **repeat**
3:     Add a new pose to X and new landmarks to L.
4:     Add the corresponding new factors to $g(\Theta)$
5:     $\Theta^* \leftarrow \arg\min_\Theta (-\log g(\Theta))$ ▷ Eq: 2
6:     **if** poses added since last minimization $> \Psi$ **then**
7:         $\{X, L\} =$ **Active Minimization** $(X, L, Z, \Theta^*, g, \lambda)$
8:     **end if**
9: **until** the robot is navigating

Incremental minimization iteratively throws away less informative landmarks while using the remaining landmarks to navigate through the environment. It is less optimal and is greedy as compared to active minimization since active minimization considers all the landmarks and poses present till that moment to make the decisions where as incremental minimization keeps throwing away landmarks and does not
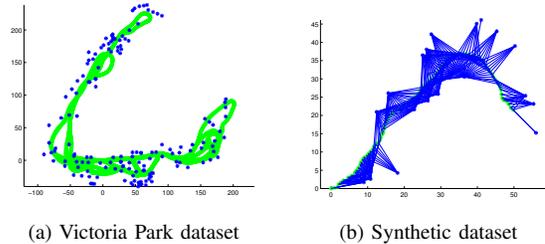

(a) Victoria Park dataset   (b) Synthetic dataset

Fig. 4: Datasets used in the experiment

	L	**P**	**Z**	**Area Covered**
Victoria Park dataset	151	6969	10608	200×250 sq. m
Synthetic dataset	23	96	422	50×50 sq. m

TABLE I: Statistics of the datasets used. **L** is #landmarks, **P** is #poses and **Z** is #measurements.

have access to all the landmarks available at a particular time. For example, incremental minimization algorithm can throw away landmark which was uninformative at time T but which might result in a loop closure constraint at $T + t$ and hence become very informative in the future. A possible solution to solve the optimality issue is to maintain a uniform distribution of landmarks over the explored region regardless of their information gain. Higher value of Ψ (calling active minimization less frequently) will result in the estimation given by incremental minimization to approach the estimation given by active minimization.

V. EXPERIMENTAL RESULTS

In this section, we describe the datasets and evaluation metrics used in the experiments and compare reduced landmark-based SLAM algorithm with the standard graph SLAM (SAM [6]) algorithm (using all landmarks and poses).

A. Datasets

We tested the reduced landmark-based SLAM algorithm on two datasets: Victoria Park dataset [11] and Synthetic dataset. Table I summarizes the datasets used and Figure 4 show the datasets. Both the datasets are planar where each pose is represented by its translation (x, y) and heading direction θ.

B. Evaluation Metrics

1) Absolute trajectory error (ATE (m)) : As proposed by Sturm et al. [22], absolute trajectory error compares the absolute distance between the trajectories estimated using standard graph SLAM and reduced landmark-based SLAM. ATE evaluates the RMSE (root mean squared error) over the difference of pose translation (for the reduced set of poses) estimated using all landmarks and poses and reduced set of landmarks and poses. Formally ATE is defined as follows,

$$ATE = \left(\frac{1}{n} \sum_{i=1}^{n} \| trans \left(X^i - X_r^i \right) \|^2 \right)^{\frac{1}{2}} \quad (8)$$

where X^i is the ith pose estimated using all landmarks and poses and X_r^i is the ith pose estimated using reduced set of landmarks and poses.

2) Absolute landmark error (ALE (m)): Similar to ATE, absolute landmark error computes the RMSE over the reduced set of landmarks.

$$ALE = \left(\frac{1}{n} \sum_{i=1}^{n} \|L^i - L_r^i\|^2 \right)^{\frac{1}{2}} \tag{9}$$

3) Average rotation error (ARE (deg)): Average rotation error is evaluated by averaging the angular difference in the pose heading directions over the reduced set of poses.

$$ARE = \frac{1}{n} \sum_{i=1}^{n} \|rot\left(X^i - X_r^i\right)\| \tag{10}$$

4) Eigen Envelope (EIG): As proposed by Carlevaris-Bianco et al. [2], we compute the range of eigen values of the difference between the marginal covariances of the reduced set of poses and landmarks when using all landmarks and poses as compared to using reduced number of landmarks and poses.

$$EIG = range\left(eig\left(\Sigma_{TRUE} - \Sigma_{REDUCED}\right)\right) \tag{11}$$

5) Percentage increase in covariance determinant of the latest pose (UD): Uncertainty of the latest pose is computed by marginalizing out the latest pose and evaluating its covariance determinant. We compute the percentage increase in uncertainty of the latest pose when using all landmarks and poses as compared to using reduced number of landmarks and poses.

$$UD = \frac{\|\Sigma_{REDUCED}^{LP}\| - \|\Sigma_{TRUE}^{LP}\|}{\|\Sigma_{TRUE}^{LP}\|} \times 100 \tag{12}$$

C. Results

We compare the two components of the reduced landmark-based SLAM algorithm, active minimization and incremental minimization with standard graph SLAM algorithm. The minimization algorithms are analyzed by comparing the pose trajectory and landmark location estimated by the reduced set of landmarks and poses as compared to the estimation by standard SLAM using all landmarks and poses. The accuracy of the solution is analyzed using the evaluation metrics given in Section V-B. In the following experiments we use $\lambda = 0.4$ as the weight factor to conserve more landmarks and $P = 5$ as the pose lag.

1) Victoria park dataset: For Victoria Park dataset, we use $\Psi = 500$ as a pose interval at which active minimization is called in an incremental manner. Figure 5 plots the estimated pose trajectories and landmark location for Victoria Park dataset using standard graph SLAM and reduced landmark-based SLAM (active minimization and incremental minimization) at three different time intervals. Table II compares active minimization and incremental minimization algorithm w.r.t standard graph SLAM using the evaluation metrics for the trajectories shown in Figure 5. Figure 6 compares active minimization and incremental minimization algorithm using the evaluation metrics at every 500th pose.

We can see from Figure 5 and Table II that the trajectory and landmark location estimated using standard graph SLAM

is similar to the estimation given by reduced landmark-based SLAM. The final absolute trajectory error between the estimation given by reduced landmark-based SLAM (incremental minimization) as compared to standard graph SLAM is 2.3 m with a 50% reduction in the number of landmarks and 54% reduction in the number of poses. The absolute landmark error is 2.05 m as well. The average error in heading direction is 0.0205 degrees. As compared to incremental minimization, active minimization uses all the landmarks and poses and therefore has better accuracy of the estimated trajectory (Table II and Figure 6). This also shows the sub-optimal nature of the incremental minimization as compared to active minimization. Another reason of high errors in case of incremental minimization as compared to active minimization is because minor trajectory errors added initially in time propagate through out rest of the estimation as it progresses. The number of landmarks and poses selected by active minimization is more than the number of landmarks and poses selected by incremental minimization. Positive values of eigen envelope (EIG) show that the estimates are conservative. Percentage increase in uncertainty of the latest pose (UD) is reasonable for both active minimization and incremental minimization algorithms.

2) Synthetic dataset: For this dataset, we use $\Psi = 30$ as a pose interval at which active minimization is called in an incremental manner. Figure 7 plots the estimated pose trajectories and landmark location for Synthetic dataset using standard graph SLAM and reduced landmark-based SLAM (active minimization and incremental minimization) at two different time intervals. Table III compares active minimization and incremental minimization algorithm w.r.t standard graph SLAM using the evaluation metrics.

We can see in Table III that the ATE and ALE of the estimation given using reduced landmark-based SLAM as compared to using standard graph SLAM is 0.2231 m and 0.3365 m with a 39% reduction in the number of landmarks. Since after removing uninformative landmarks there is no pose that doesn't see any landmark, we do not marginalize out any pose and therefore there is no reduction in the number of poses. The average error in the heading direction is 0.0158 degrees. For the synthetic dataset the trajectory estimated using active minimization is the same as the trajectory and landmark location estimated using incremental minimization since there are no loop closures. In the presence of loop closures, landmarks which are less informative at certain point of time can become informative in the future. Percentage increase in the uncertainty of latest pose (UD) is low for active minimization algorithm (0.08%), where as it is high for incremental minimization algorithm (16.9%) as compared to full SLAM. This is due to sub-optimal removal of landmarks in the case of incremental minimization.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we developed an information theoretic algorithm to efficiently reduce the number of landmarks and poses without compromising the estimation accuracy. We proposed an incremental version of the algorithm resulting in

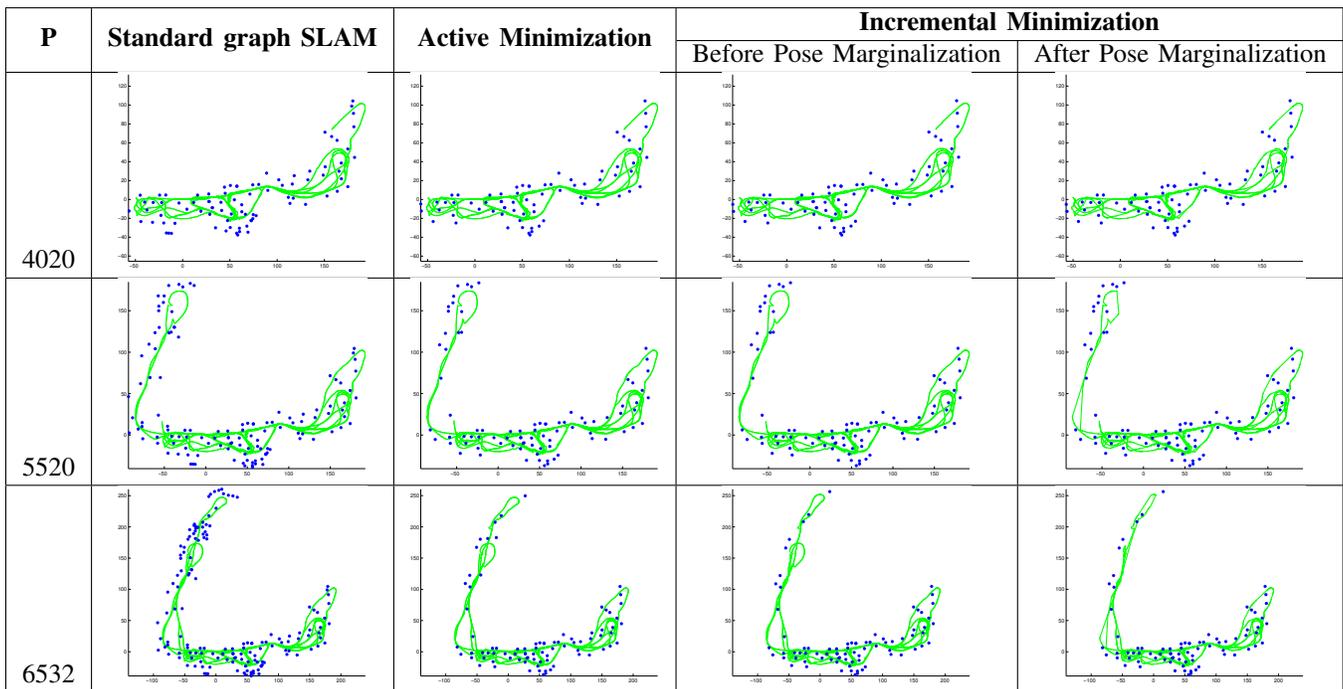| P	Standard graph SLAM	Active Minimization	Incremental Minimization	
			Before Pose Marginalization	After Pose Marginalization
4020				
5520				
6532				

Fig. 5: Estimated pose trajectory and landmark location for **Victoria Park** dataset using standard graph SLAM and reduced landmark-based SLAM at four different time intervals. **P** is #Poses. The algorithms compared here are **standard graph SLAM** (using all landmarks and poses), **Active minimization** (Alg: 3) and **Incremental minimization** (Alg: 5).

L	P	Active Minimization							Incremental Minimizaton					
		L [%]	P [%]	ATE (m)	ALE (m)	ARE (deg)	UD (%)	EIG	L [%]	P [%]	ATE (m)	ALE (m)	ARE (deg)	UD (%)
29	532	25 [13%]	308 [42%]	0.0038	0.0073	8.7185e-05	2.37e-03%	2.4e+07	25 [13%]	308 [42%]	0.0038	0.0073	8.7e-05	2.3e-03%
68	2532	58 [14%]	1345 [47%]	0.0000	0	0.0000	1.37%	4.5e+09	58 [14%]	1345 [47%]	0.1027	0.1461	0.0052	3.42%
82	4532	63 [23%]	2359 [48%]	0.0207	0.0082	3.1115e-04	4.46e-02%	-	64 [22%]	2363 [48%]	0.2262	0.4196	0.0086	2.85%
139	6532	74 [46%]	2971 [54%]	0.0040	0.0044	8.3788e-05	2.07e-02%	-	70 [49%]	2958 [54%]	2.3487	2.0539	0.0205	4.14%

TABLE II: Statistics of Minimization algorithms as compared to Standard Graph SLAM for **Victoria Park** dataset. **L** is #landmarks. **P** is #poses. **ATE** is absolute trajectory error, **ALE** is Absolute landmark error, **ARE** is average rotation error, **UD** is percentage increase in covariance determinant of last pose and **EIG** is eigen envelope. **[%]** is percentage change.

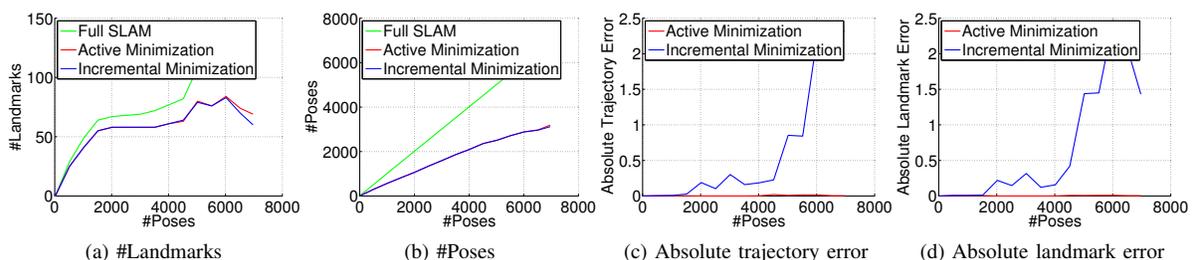(a) #Landmarks (b) #Poses (c) Absolute trajectory error (d) Absolute landmark error

Fig. 6: Comparison minimization algorithms w.r.t varying number of poses for **Victoria Park** dataset. The algorithms are compared on the basis of number of landmarks and poses remaining after minimization, absolute trajectory error (Section V-B.1) and absolute landmark error (Section V-B.2).

L	P	Active Minimization							Incremental Minimizaton					
		L [%]	P [%]	ATE(m)	ALE (m)	ARE(deg)	UD (%)	EIG	L [%]	P [%]	ATE(m)	ALE(m)	ARE(deg)	UD (%)
6	32	5 [16%]	32 [0%]	0.0815	0.0741	0.0072	40.8%	232.0	5 [25%]	32 [0%]	0.0815	0.0741	0.0072	40.8%
15	62	15 [0%]	62 [0%]	0.00	0.00	0.00	0.00%	2865.0	14 [12%]	62 [0%]	0.1108	0.1463	0.0064	0.320%
23	96	18 [22%]	96 [0%]	0.0329	0.0304	0.0035	0.08%	10018	14 [39%]	96 [0%]	0.2231	0.3365	0.0158	16.9%

TABLE III: Statistics of Minimization algorithms as compared to Standard Graph SLAM for **Synthetic** dataset. **L** is #landmarks. **P** is #poses. **ATE** is absolute trajectory error, **ALE** is absolute landmark error, **ARE** is average rotation error, **UD** is percentage increase in covariance determinant of last pose and **EIG** is eigen envelope. **[%]** is percentage change.

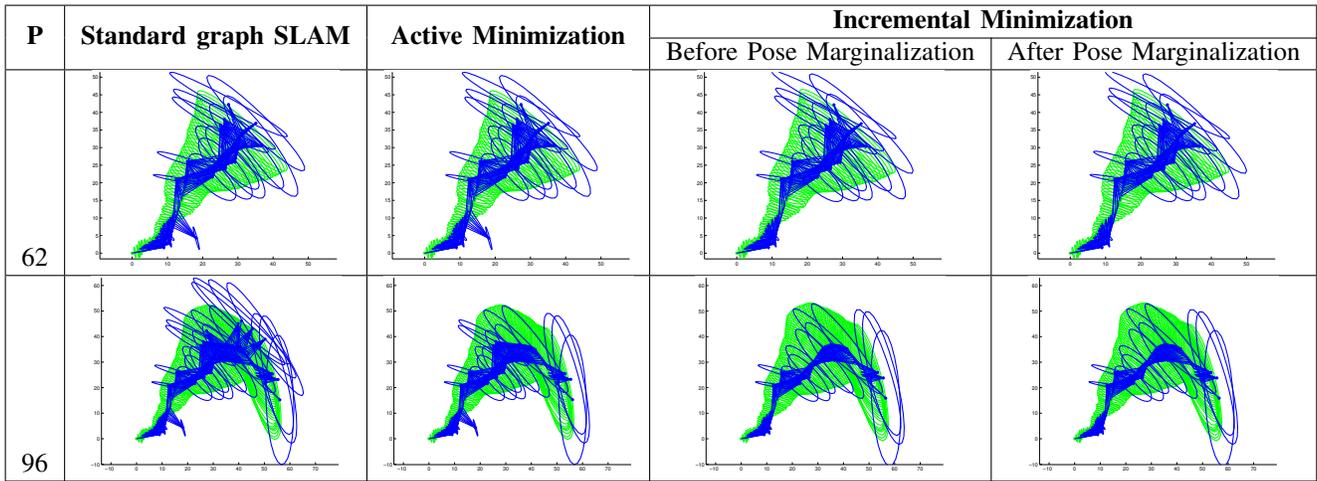| P	Standard graph SLAM	Active Minimization	Incremental Minimization	
			Before Pose Marginalization	After Pose Marginalization
62				
96				

Fig. 7: Estimated pose trajectory and landmark location for **Synthetic** dataset using Standard Graph SLAM and reduced landmark-based SLAM for different trajectories. **P** is #Poses. The algorithms compared here are **Standard Graph SLAM** (using all landmarks and poses), **Active minimization** (Algorithm 3) **and Incremental minimization** (Algorithm 5). Green ellipses represent pose covariances. Blue ellipses represent landmark covariances.

reduced landmark-based SLAM algorithm. We showed the results on Victoria park dataset and Synthetic dataset and compared trajectory and landmark locations estimated using standard graph SLAM (SAM [6]) and reduced landmark-based SLAM algorithm showing a reduction of 40-50% landmarks and around 55% poses with minimal estimation error as compared to standard SLAM algorithms.

As a future work, we are working on a parallel minimization and mapping algorithm where active minimization can be run in parallel to the standard graph SLAM algorithm. Another possible future direction is to improve the active minimization algorithm (Algorithm 3) by considering a cluster of landmarks instead of one landmark at a time [15].

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